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ANALYSIS OF THE CASH RECOVERY RATE'S
POTENTIAL APPLICATION IN DETERMINING
THE SOCIAL COST OF CAPITAL

THESIS

Francis J. Geiser, III, Captain, USAF

AFIT/GSM/LSY/91S-11

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ANALYSIS OF THE CASH RECOVERY RATE'S POTENTIAL
APPLICATION IN DETERMINING THE SOCIAL COST OF CAPITAL

THESIS

Presented to the Faculty of the School of Systems and
Logistics of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Systems Management

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September 1991

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Preface

The purpose of this study was to contribute to the existing empirical evidence supporting the use of the cash recovery rate as a surrogate of the internal rate of return. Validation of such a relationship would provide economists with the much needed ability of externally measuring firm profitability. Once obtained, these measures could become the basis for the formulation of a social discount rate derived from average costs of capital characterizing each sector of the economy. A glossary of technical terms is provided in Appendix A.

I would have not been able to complete this thesis if it were not for the professional guidance and personal support of others. I commend my faculty advisor, Major David S. Christensen, for his exceptional ability to lead me through this research process. I thank him for sharing his patience and insights, and most of all, for not discouraging my ambition to undertake what has turned out to be the most innovative learning experience of my graduate program. I wish to thank my wife Terri and my children, John, Anthony, and Vincenza. Unlike mine, their lives have not been on hold for the last 15 months. They have struggled with the day to day challenges, often without my help, and still found time to make my days and nights most comfortable.

Francis J. Geiser, III

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Abstract

This study explored the potential application of the cash recovery rate in determining the social cost of capital. It specifically investigated the results of using cash recovery rate-based, internal rate of return-estimating relationships, which were formulated under assumptions of constant investment growth rate, to estimate internal rates of return for simulated firm-level financial data generated using both exponentially increasing and sinusoidal investment growth rates. An extensive literature review is used to build a analytical link between the need for updating the social discount rate, capital budgeting decisions based on internal rates of return, and the proposed behavior of the cash recovery rate as a surrogate for the internal rate of return. Analysis of the estimated internal rates of return, based on an algorithm developed by the author, indicates the relationships remain valid under the more complex growth rate patterns. This result further supports the use of cash recovery rates as a means of measuring firm profitability and also strengthens the case for additional research into applying an opportunity cost of capital approach for the selection of a social discount rate based on a nationally averaged internal rate of return.

ANALYSIS OF THE CASH RECOVERY RATE'S POTENTIAL
APPLICATION IN DETERMINING THE SOCIAL COST OF CAPITAL

I. Introduction

Background

"Few topics in (the) discipline (of economics) rival the social rate of discount as a subject exhibiting simultaneously a very considerable degree of knowledge and a very substantial level of ignorance" (Baumol, 1968:788). The report from the Subcommittee on Economy in Government, published in 1968, clearly embodied congressional concern with then contemporary practices of discounting (see Appendix A) the future benefits of public investment alternatives (Subcommittee, 1968:7). Government agencies had established the use of a discount rate (DR) (see Appendix A), routinely referred to as the social discount rate (SDR) (see Appendix A), to equate the dissimilar future cash flows of competing proposals. These discounted cash flows (DCF) (see Appendix A) became the foundation for a systematic solution to the problem of choosing how to best use scarce fiscal resources.

During the interim period since these congressional hearings, notable public sector economists, including William J. Baumol, David F. Bradford, J. A. Seagraves, and

Jacob Stockfish, have perpetuated the seemingly unresolvable disagreement over exactly what the SDR is meant to represent and how it should be derived. "Roughly speaking, the division within the profession is threefold" (Tresch, 1981:486). One group believes the SDR should reflect an opportunity cost of capital (OCC) (see Appendix A) equal to the value that could have been earned if the public investment funds were left in the private sector. A second group asserts the SDR indicates society's willingness to forgo present consumption in lieu of benefits for future generations. The third opinion supports computing the SDR as a weighted sum of the first two techniques. Most theoretical models used to evaluate the present value (see Appendix A) of public expenditures incorporate all three considerations, but vary substantially over the emphasis placed on each of the factors.

Seagraves, an advocate of the OCC approach, defended a single criterion, the internal rate of return (IRR) (see Appendix A), with which government agencies could evaluate competing projects. Though it was not a panacea, he was able, because of the IRR approach, to quantify characteristics considered indeterminate in many of the empirical discounting models under investigation. Unfortunately, his work was stagnated by the elusive nature of IRR measurements.

In a series of articles between 1978 and 1980, Yuji Ijiri introduced his hypothesis that the IRR could be

estimated from the less obscure cash recovery rate (CRR) (see Appendix A) (Ijiri, 1978; Ijiri, 1979; Ijiri, 1980). Gerald L. Salamon and Ijiri have since formulated numerical models calculating IRRs based on the knowledge of CRRs (Ijiri, 1978; Ijiri, 1979; Ijiri, 1980; Salamon, 1982; Salamon, 1985; Salamon, 1988).

A cross-sectional review of contemporary capital budgeting literature reveals a growing reliance on IRRs for the cost-benefit analysis of potential investments (Anonymous, 1985; Blazouske and others, 1988; Hellings, 1984; Moore and Chen, 1984; Newbery and others, 1990). Acceptance of the argumentation posed by Seagraves and other proponents of an OCC-based SDR, combined with the suggested utility of CRRs, forms the foundation of an innovative proposition for updating the selection of the SDR based on a nationally-derived, CRR-estimated IRR.

Justification

Robert Shishko, an economist in the Rand Corporation, suggests,

The ultimate impetus for this research (DR) . . . derives from the fact that many decisions within the Department of Defense (DoD), involving billions of dollars of public funds, could be improved if the meaning of the discount rate and the uses of discounting were better and more widely understood. (Shishko, 1976:iii)

The DR is a critical factor in decision-making when evaluating commensurable proposals strictly on an economic basis. Consider the following example (see Figure 1).

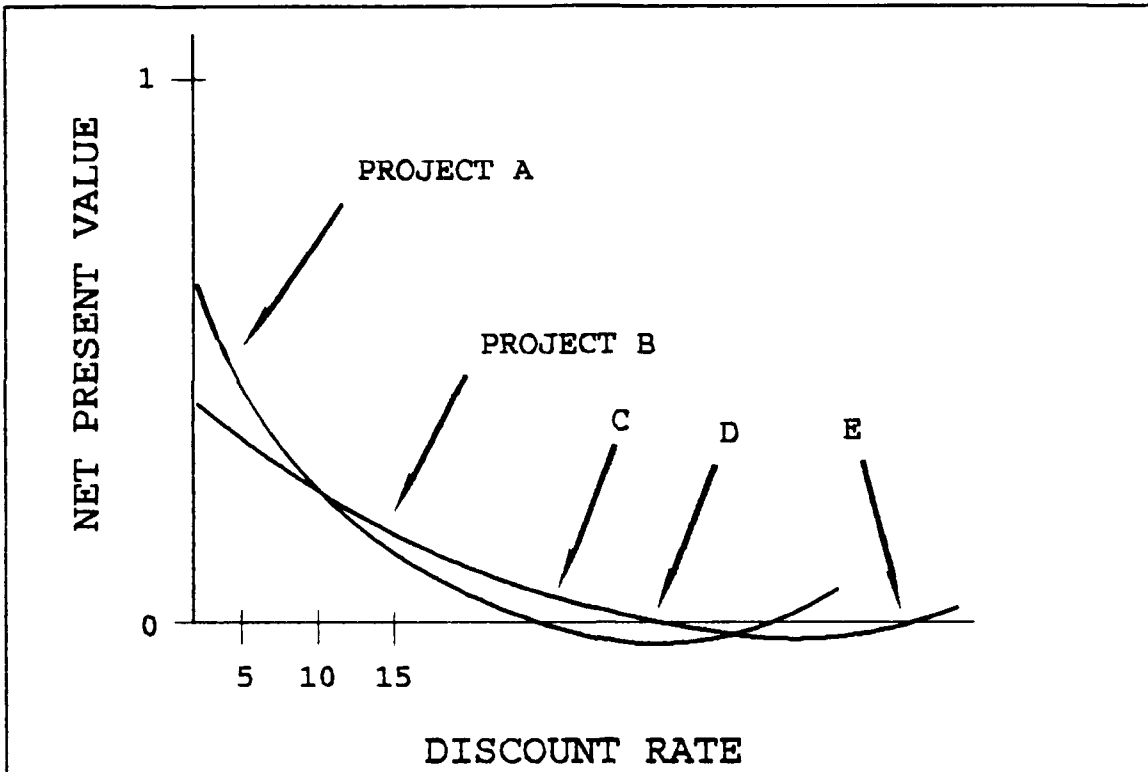


Figure 1. Sensitivity of Net Present Value to Discount Rate

When the net present value (NPV) (see Appendix A) of both investments is discounted at a SDR of 15 percent, Project B's NPV is greater than Project A's NPV. Project B appears as the optimal economic choice. Now, consider a five percent SDR. This change evokes two notable phenomena. By decreasing the SDR, Project A becomes the most cost-effective investment. Even more significant than the reversal in NPV ranking, the smaller SDR implies selecting the alternative with the lower IRR, point C on the curve for Project A versus point D on the curve for Project B.

The need for discounting arises in the evaluation of public investment because the varied behavior of costs and

benefits occurring throughout the life spans of different projects must be reduced to a single array of values referenced to one preselected time frame. Almost all economists agree that discounting is the best technique to convert a comprehensive collection of costs and benefits into a single format for the purpose of meaningful comparison. However, there is no consensus on the derivation of the DR necessary to accomplish such transformations. Discord concerning the appropriate discounting process is not the only controversy.

Robert Shishko acknowledged the need to scrutinize and possibly update SDR selection methodologies when he declared many public "economists harbor the suspicion that numerous government projects that would be rejected by the private sector are funded because the wrong discount rate is used . . ." (Shishko, 1976:v). Since 1976, there has been considerable progress in understanding discounting and the implications of predicaments worthy of the skepticism identified by Shishko. Despite these gains,

the problems facing policy analysts today are possibly more complex than in the past. There still exists confusion as how best to apply the improved understanding of the conceptual issues to practical usage. (Quirk and Terasawa, 1987:vii)

Throughout these changes, one aspect remains incessant.

"The stakes are large, since the discount rate is typically the most crucial factor in determining project acceptability" (Wright and Warr, 1990:240).

Several economic studies completed between 1958 and 1975 support the use of DRs ranging from 6 to 12 percent. It is certainly possible the paradox described in the varying DR example above would result from allowing the DR to fluctuate over this range. The studies of Jacob Stockfish and Arnold Harberger, considered the two best estimations of the DR by Shishko, base the derivation of the SDR on the rate of return of the corporate and personal sectors of investments (Subcommittee, 1968:12; Shishko, 1976:26; Stockfish, 1969:2+). These studies advocate DRs of 10.67 and 8.33 percent, respectively.

In 1972, the Office of Management and Budget (OMB) directed most federal agencies to apply a SDR of 10 percent in NPV calculations. The United States Air Force (USAF), following the OMB's guidance, uses the 10 percent DR when performing cost-benefit analysis of competing investment proposals. Additionally, Air Force Regulation (AFR) 173-15 endorses a DR of five percent as the lower bound to be used in sensitivity analyses. No upper bound has been declared. Determination of an upper bound for DoD cost-benefit sensitivity analyses represents a credible application for a SDR derived from national-level IRRs. Analytic support for this assertion will be described in detail in Chapter II of this thesis.

Yuji Ijiri, a strong proponent of cash-flow accounting, proposed the cash recovery rate

is equal to the discounted cash-flow rate (see Appendix A) if the project has an infinite life, and an excellent approximation of the (IRR) if the recovery rate is over 15 percent and the project life is over 15 years, which is perhaps the case in a majority of capital investment decisions. (Ijiri, 1980:55)

Ijiri suggested DCFs replace the earnings-based criteria commonly applied in evaluating firms' economic performance (see Appendix A). Both CRRs and IRRs focus on cash flows. Studies have shown,

the (CRR) for many corporations is remarkably stable from year to year, suggesting that the average profitability of corporate projects can be measured through CRRs. (Kaplan, 1982:547)

Assumptions about the average life of projects can be added after the CRR is computed. This denotes a favorable departure from the limitations of conventional performance appraisal (Ijiri, 1980:54+; Kaplan, 1982:548).

Though economists choose many separate paths in the derivation of the social cost of capital (SCC) (see Appendix A, Social discount rate), most embrace the concept of DCFs as a common point of embarkation. Ijiri's CRR propositions introduce an opportunity to improve investment-performance ratings (which result from using cash flows instead of return on investment as the evaluation criterion) and simplify the estimation of IRRs. These two contributions, viewed from the perspective of discounting government investments, support the exploration of the potential application of CRR theories to the task of upgrading SCC evaluations.

General Issue

Some economists suggest present public discounting policies, as directed by the OMB, used to evaluate equally capable proposals on a cost-benefit basis are outdated (Quirk and Terasawa, 1987:46). Revisions to the SDR would affect defense acquisitions costing billions of dollars. In 1970, economist J. A. Seagraves recommended the selection of government investments based on their predicted IRRs. Economists Gerald L. Salamon and Yuji Ijiri have suggested the more readily measured CRR is a surrogate for the IRR. Current numerical models used to estimate IRRs based on CRRs must be validated under more complex and realistic scenarios than those already examined (Brief, 1985:473+; Gordon and Hamer, 1988:514+; Salamon, 1982:292+; Stark, 1989:277+).

Specific Problem

There are two distinctive obstacles to the theoretical application of CRRs in the calculation of a SDR. The first issue involves validating the theory's assumptions concerning the cash-flow parameter associated with the firms under investigation.

The second hurdle, which can be accomplished independent of the first, entails further verification of the CRR's ability to approximate IRRs. Computational models used to calculate CRRs are based on simplified economic assumptions. One of the principal assumptions constrains firms' investment growth rate to a constant value (Ijiri,

1980:54+; Salamon, 1982:292+). Skeptics have challenged the models' dependency on this condition, which they consider impractical (Lee and Stark, 1987:125+; Stark, 1987b:99+).

The purpose of this research is to extend the validation of CRR-derived estimations of IRRs. Positive correlation between IRRs estimated by empirical models operating under more complex financial conditions and actual economic rates of return would denote a pivotal improvement to the external estimation of firms' economic performance. Accurate measures of firm profitability would contribute to the selection of a SDR or the determination of a DR to serve as the upper limit in cost-benefit sensitivity analyses.

Hypothesis and Investigative Questions

Research Hypothesis. Under the condition of a variable annual investment growth rate, cash recovery rate-derived estimates of internal rates of return will equal the corresponding, directly calculated internal rates of return. The following criteria will be used to evaluate the hypothesis:

Investigative Questions. 1. Do the numerical models estimate the IRR adequately to support the postulate of the CRR being a surrogate for the IRR?

2. Is there a correlation between the magnitude of the percent error (see Appendix A) in the IRR estimates and any of the parameters used to calculate these CRR-derived IRRs?

3. If a correlation exists, is it significant enough to predict when the models' inaccuracies may exceed an acceptable percent error level?

Scope and Limitations

This research is not intended to solve the enduring opposition among the myriad of respected and acclaimed economists concerning the endorsement of any singular approach to the determination of a SDR. Chapter II's review of the literature will develop an understanding of the multifarious debate over what a SDR is meant to represent. Focus will shift to the introduction of the IRR as a candidate for the SDR and a sampling of the IRR's expanding use in contemporary capital budgeting decisions. The main emphasis will center around the development of CRR theory, its empirical models, and the conclusions and challenges of past analytical efforts.

Many assumptions made by Ijiri, Salamon, Stark, and others in their numerical modeling of the CRR await validation. This research will analyze only the assumption of constant investment growth rates. The numerical models will be mathematically modified to accept variable input for this parameter. All analysis will be performed using simulated cash flows.

Summary

The uncertainty of accuracy related to the SDR has disturbed those involved in public investment for quite a long time. Considering the billions of dollars spent for new

projects by the DoD alone, it is obvious the apprehension will persist. The intent of this research is to substantiate the unprecedented coupling of IRRs, predicted from CRR calculations, with the derivation of a SDR. With these tools, the public sector may be better equipped to minimize the SCC resulting from government expenditures.

II. Literature Review

Introduction

This literature review presents background information in support of the postulate that a national-level IRR, derived from a CRR of comparable breadth, would significantly contribute to the understanding and use of discounting when determining the SCC. Economic issues to be covered include the following topics: a) the polemic debate concerning the meaning and selection of a SDR; b) the use of IRRs in capital budgeting; and c) the development and maturation of CRR theories.

Scope and Limitations of the Literature Review

Yuji Ijiri, in 1978, first introduced his, " . . . cash recovery ideas as exploratory . . . , " and voiced a request, " . . . for further thinking in the area" (Lee and Stark, 1987:125). Several economists have published conclusions concerning the validity of the CRR as an instrument for earnings-based performance measurement, the accuracy of CRR approximations of the IRR, and the compatibility of CRRs and capital budgeting (Brief, 1985; Gordon and Hamer, 1988; Ismail, 1987; Lee and Stark, 1987; Salamon, 1982, 1985, 1988; Stark, 1987a, 1987b, 1989). This literature review will concentrate on sources addressing the feasibility of developing the CRR as a tool for investment and performance decisions and its potential as a surrogate for the IRR.

A broad survey of SCC articles provides the reader with an understanding of the continual controversy surrounding applications of the SDR without trying to resolve this debate. Though pertinent theoretical and empirical analyses will be referenced, no attempt will be made to support the proof or mathematical derivation of economic principles and laws.

Method of Treatment and Organization

This literature review begins with a brief review of discounting concepts used in capital budgeting. This introduction is presented as a lead into the SDR discussions. After exploring the lasting SCC debate, this section concludes with the discounting methods presently required by the Federal Government and the DoD.

The focus will then shift to the uses of IRRs, such as Seagraves' methodology for ranking public investments, in capital budgeting. Parallel to Seagraves' proposed use of the IRR, there will be a synopsis of contemporary practices by private industries that supports cost of capital (COC) (see Appendix A) analyses based on projects' proposed IRRs.

Finally, the emphasis will center around the primary objective of studying CRR-IRR relationships and their numerical models. CRR theory will be traced chronologically through its stages of development, the challenges to its use as a measure of firm profitability, and its potential contributions to capital budgeting.

Discounting

Discounting is an economic tool used to transform the collection of future costs and benefits of a particular project into a time-period dependent function. By applying this methodology, the cost-benefit streams of alternative investments, grouped according to an equivalent level of capability, can be compared against a common selection criterion. The most economically favorable alternative, based on a predetermined reference point in time, can then be identified. Accurate discounting is directly coupled with the derivation of a relevant DR.

When considering discounting procedures and evaluating their use by the Federal Government, specifically by DoD, the reader must maintain a categorically defined frame of reference. Discounting is a means of rearranging cost streams for equally effective investments and not a decision model to determine the feasibility or necessity of the proposed projects. The very nature of the economy and the highly imperfect capital market motivate the wide range of concepts offered as the basis for a suitable SDR.

Social Discount Rate

Early Discussions. (Quirk and Terasawa, 1987:3-6)

The issue of the optimal discount rate to use in evaluating government projects is one that has been debated in the economics literature since the late 1950s, when it arose in connection with cost-benefit studies of water projects.

In 1928, D. Ramsey first introduced the issue of the ethical justification for discounting the utilities of future generations in his essays on the optimal savings rate for society and intergenerational equity. Though his recommendations have not played a major role in the development of public investment policies, the questions he raised remain central to the discussion of choosing of a SDR.

In the early 1960s, A. Sen and S. Marglin revived the arguments first posed by Ramsey.

Meanwhile, a voluminous literature developed dealing with one-, two-, and n-sector growth models, including the optimal growth model of Cash, in the Ramsey tradition.

K. Arrow, in 1966, "specifically formulated the problem of choice of the social discount rate as one of determining the optimal growth path for an economy." Arrow's approach has been followed in most of the formal SDR literature since its introduction. However, it was a

paper by W. Baumol (1968) that seems to have provided the spark for the debate that developed during the 1970s over the social rate of discount. Baumol views the choice of the social rate of discount as one to be made on opportunity cost grounds.

Contemporary Issues. Many economists share one of three prominent views on how the numerical value of the SDR should be obtained: a) from an opportunity cost approach; b) from a time preference approach; or c) a combination of both strategies. Several economists, including D. Bradford, P. Diamond, and R. Lind, contend that use of the time

preference approach must include considerations for the shadow price of capital (see Appendix A). Debates over incorporating the impacts of risk and uncertainty are tightly woven into each of the strategies.

The first view contends the SDR should reflect an OCC, also known as the inter-temporal marginal rate of transformation (MRT) (see Appendix A) (Baumol, 1968:788; Shishko, 1976:4). The slope of Curve A in Figure 2 graphically tracks the MRT.

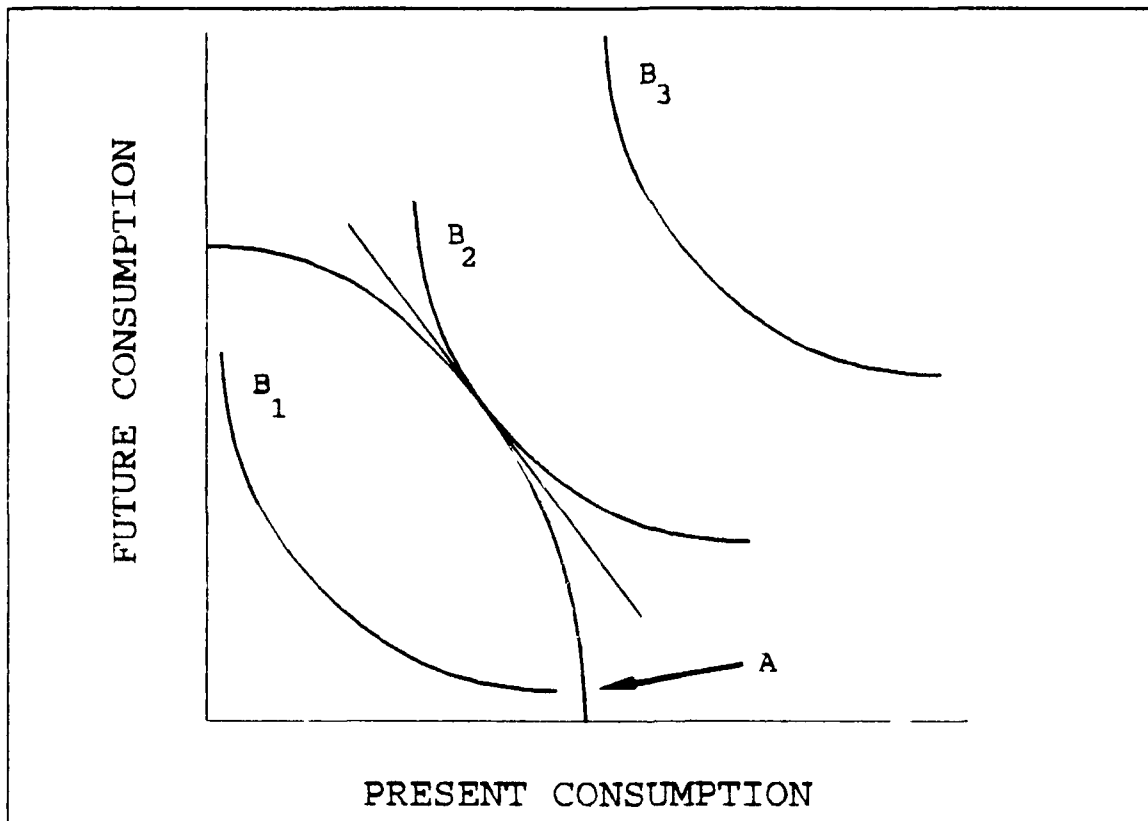


Figure 2. Intertemporal Marginal Rates

Baumol stated the SDR should measure "the opportunity cost of postponement of receipt of any benefit yielded by public investment." The components making up this measure (rate) should include

primarily the welfare foregone by not having these benefits available for immediate consumption or reinvestment and (perhaps) a premium corresponding to the risk incurred in undertaking government projects. (Baumol, 1968:788)

Baumol's perspective categorizes what the government takes from the private sector as input resources. "To determine the relevant rate of discount, one need not inquire beyond the rate of return currently being earned by the users of such inputs" (Baumol, 1968:792).

There have been a number of studies that measured the SDR. "The most widely acknowledged of these are separate studies by Harberger, Haveman, and Stockfish" (Shishko, 1978:2). A. Harberger calculated the SDR as a weighted average of the after-personal-income-tax rate of return to savers and the pre-corporate-income-tax COC. R. Haveman used the assumption that additional government revenue will be financed completely through personal income taxes when he calculated the SDR as a weighted average of various consumer borrowing rates.

J. A. Stockfish estimated the SDR by computing the marginal productivity of capital in the private sector. His estimate was calculated as the weighted average of the pre-tax rate of return in several corporate sectors (manufacturing, transportation, and public utilities) with

the rate of return in the noncorporate sector (agriculture and non-farm unincorporated business). He estimated 70 percent of corporate capital earned 16.5 percent dividends and 30 percent earned 11.5 percent dividends before taxes and inflation. This resulted in an average of 15 percent. The noncorporate sector's portion of total capital was assumed to earn 10 percent. Weighting the corporate and noncorporate sectors 40 percent and 60 percent, respectively, and then subtracting 1.6 percent as the average rate of inflation for the years 1949-1965, Stockfish obtained a real SDR of 10.4 percent (see Table 1 for SDRs from various studies). (Stockfish, 1969:193)

J. A. Seagraves noted expanding government activity reduces private investment and increases interest rates. The increasing interest rates lead to additional private savings. Contrary to Stockfish's use, Seagraves claims,

Only if savings did not respond to greater government borrowing would it be legitimate to equate the (SDR) with the marginal productivity of capital. (Seagraves, 1970:44)

Seagraves based his choice of the SDR on the OCC from the private sector with considerations for risk, taxes, and inflation. Seagraves agrees with Baumol's and Arrow's views that any one project presents negligible risk to the public spending, but suggests the SDR should include an adjustment for risk. This risk premium would provide the opportunity for private sector investment in high risk, high rate of return projects otherwise taken by the government. Because

TABLE 1
MEASUREMENTS OF THE SOCIAL DISCOUNT RATE
(Seagraves, 1970:448; Shishko, 1978:7)

Author	Year ^a	Recommended Nominal Rate (%)	Adjusted for Expected Inflation ^b (%)
Hirshleifer, DeHaven and Milliman	1958	6.00	4.58
Haveman	1966	7.30	5.95
DoD Directive ^c	1966	---	10.00
Stockfisch ^d	1949-1965	12.00	10.67
Harberger	1968	10.68	8.33
Baumol	1968	10.00	7.65
Seagraves ^e	1969	14.50	12.15
OMB Directive ^f	1972	---	10.00

Note: ^athe year rate applies; ^brate is a geometric average of the inflation rate for six year period prior to year shown; ^cDoD Instruction 7041.3, December 19, 1966; ^drate calculated for full period shown is 10.4; ^eoriginally reported as 11.5 using 3 percent for inflation; and ^fOMB Circular No. A-94, March 27, 1972.

Seagraves uses the yield on corporate bonds as the foundation for the SDR, he includes a factor to compensate for corporate profits and other taxes private firms must pay. After determining the marginal productivity of capital, Seagraves included an adjustment for the resultant increase in private savings. He then subtracted a percentage for expected inflation. Calculation of his real SDR for the beginning of 1969 is shown in Table 2 (Seagraves, 1970:430+). Each of the studies mentioned in

TABLE 2

SEAGRAVES' SOCIAL DISCOUNT RATE (Seagraves, 1970:448)

Basic Factors Affecting the Social Discount Rate	Social Discount Rate	
	Lower Limit (%)	Upper Limit (%)
Yield on Class A corporate bonds	6.7	7.2
Risk premia for government portfolios	+2.0	+4.0
Corporate profit and property taxes	+4.3	+6.0
Marginal productivity of capital	13.0	17.2
Adjustment for added savings	-1.5	-1.5
Social rate of discount in money terms	11.5	15.7
Adjustment for expected inflation	-3.5	-1.5
Social rate of discount in real terms	8.0	13.2

the preceding text assumed the SDR is a measurable entity; this prerequisite is a main concern for economists not supporting the MRT approach (Shishko, 1978:2).

Many economists suggest there exists significant constraints relating to OCC measurements. They also suggest, "When the private investment opportunities are not also government investment opportunities, private rates of return become irrelevant to government choices" (Bradford, 1975:888). The second major SDR view uses the social rate of time preference, also known as the intertemporal marginal rate of substitution (MRS) (see Appendix A) as the deciding factor in establishing the DR for public ventures. The slope of the consumer indifference curves $B_{(1-3)}$ in Figure 2 graphically represent the MRS. Conceptually, the MRS

denotes a rate of return set to a level that will induce consumers to save now for future consumption.

DRs based on the MRS usually are lower than those calculated by MRT methodology; MRS antagonists charge such an approach often suggests the funding of unfavorable projects or investments whose net benefits occur farther and farther in the future. Baumol likens the use of a MRS-based SDR to a "Robin Hood activity stood on its head" (Baumol, 1968:800). He denounces the use of a traditionally lower rate derived from the MRS on the grounds that it favors investments which forfeit present consumption in support of future generations "whose per capita income will likely be a sizeable multiple of its present value" (Baumol, 1968:800).

Some economists suggest society's MRS can be derived from household decisions regarding savings, consumption, borrowing, and lending. Others submit the MRS is a function of the type project being considered and society's associated attitude. A third group proposes the rate can be inferred from past "voter-consumer referenda by whether (a class of projects) was accepted or rejected." Despite the varied opinions, there seems to be a consensus among public sector economists that rates of return between 3 and 6 percent reflect consumer's MRS (Tresch, 1981:489). (Shishko, 1976:3-8)

In an ideal world with no taxes or market imperfections, the MRT and MRS will be equal (see Figure 2, curve B_2). In the U.S. economy, the MRT and MRS are driven

apart primarily by differences between the corporate and consumer tax systems. In recognition of the irreconcilable arguments, several economists, including Arrow, Bradford, Diamond, Kurz, Lind, Stiglitz, and Usher, support modeling of the SDR as a compromise between the two approaches. (Quirk and Terasawa, 1987:24)

Weighted-average discounting, first suggested by Diamond in 1968 and Harberger in 1972, has been frequently restated throughout the public policy literature as a valid technique to assess the SDR in a second-best world (see Appendix A). Many of these models, which exploit the overlap between the MRT and MRS, appear to imply the SDR is biased towards the MRT (Tresch, 1981:496).

David Bradford developed a second-best model which concluded the true SDR is probably closer to the consumer's MRS. His hybrid approach determined the appropriate SDR in an economy where the government has certain investment opportunities that are not available to the private sector. (Bradford, 1975:896+; Tresch, 1981:491+). His main objective was to explore the relevance of Arrow's conclusion that

if capital market imperfection takes the form of a fixed marginal propensity to save out of private income (independent of the rate of return), the optimal government investment policy in the long run is to invest to the point where the marginal rate of return on government capital equals the marginal rate of social time preference (whether or not derived from individuals' rates of time preference), regardless of the rate of return on private capital. (Bradford, 1975:887)

Bradford begins with a simple model based on three assumptions: a) the private investment-discouraging and -encouraging effects of public investment are at the same rate per dollar; b) the shadow price of capital is constant and greater than one; and c) all yield of investments occur in the period immediately following the investment. The model was then expanded to include variable shadow prices and multi-period yields. Bradford tested four conditions: a) (the simple model's first assumption); b) public investment induces no private investment (shadow price approaches zero); c) all yield is converted into private investment (shadow price approaches infinity); and d) there is no yield in either government or private investment. The resultant SDRs of these scenarios are equal to the MRS, greater than the MRT, less than the MRS, and equal to a weighted average of the MRS and MRT with the weighting being the proportions in which the resources were taken out the public and private sectors, respectively. Bradford points out

the solution generally lies, interestingly enough, not on either horn of the dilemma, time preference or private productivity discounting, and may not even lie between the two apparent extremes.
(Bradford, 1978:892-893)

He submits in conclusion a SDR based on the MRS responds rather robustly to the varying parameters underlying the second-best criterion but warns about being optimistic that one rate can be applied to any government investment and lead to a correct choice. (Bradford, 1975:887+)

Quirk and Terasawa offer the following summarization to be more or less a consensus view concerning risk and uncertainty:

Except for extraordinary cases, the social rate of discount on a risky government project should be the private risk-free rate of return on a comparable project, and net benefits of any project should be evaluated on the basis of their expected values. The extraordinary cases are cases in which the net benefits are strongly correlated with national income; if the correlation is strongly negative, the social discount rate for the project should be less than the private risk-free rate, and if the correlation is strongly positive, then the social rate should be greater than the private risk-free rate. (Quirk and Terasawa, 1987:43-44)

This consensus directly reflects the arguments of pooling and the spread of returns from government projects being distributed independently of national income developed by the Arrow-Lind Theorem (Tresch, 1981:508+).

Continual Refinements. Over the years, many improvements have been made to the second-best models. Advancements include considerations for the techniques used to fund public investment, the interrelationship between public and private production, the level of commodities offered by the government, the optimization of shadow prices, multiperiod investment as well as yield, and the extension to a third-best world model (Auerbach, 1987; Newbery, 1990; Quirk and Terasawa, 1987). Many of these new approaches reaffirm the conclusions of past models while others suggest new avenues for the selection of a SDR.

A. Auerbach defended the weighted-average approach to public discount rates as being valid in a variety of circumstances regarding the availability of financing and investment instruments to the government in either period of a two-period model. His determinations of the SDR, which depend on the conditions constraining the government, are derived from the MRT, the MRS, a weighted average of the two, or the ratio of shadow prices from each of the periods in the model. (Auerbach, 1987:40+)

Quirk and Terasawa propose the SDR's task of filtering government projects is a third-best situation and the DR needed is what they term the government opportunity cost rate (GOCR). The third-best world reflects the reality of a fixed level of government spending as the result of political and fiscal policies. This contrasts the second-best model choosing the level of government spending which optimizes social welfare. Selection of the GOCR optimizes the portfolio of projects funded within a given budget.

At a steady-state optimum, the GOCR of return is the maximum rate of return that can be earned from investing an additional dollar in the portfolio of unfunded projects, subject to the constraint that the aggregate investment expenditure associated with the portfolio of funded projects is equal to the amount of investment funds available under the predetermined budget. (Quirk and Terasawa, 1987:33)

It is interesting to note that if the necessary assumption of fungibility (see Appendix A) in the application of the third-best model is violated, the GOCR becomes the MRS. (Quirk and Terasawa, 1987:32-40)

The shadow price of capital approach has received considerable interest lately precisely because it suggests a resolution to the dilemma resulting from unequal rates of opportunity cost and time preference. (Lyon, 1990:S-38)

This approach is based on the distinction between the values of the shares of costs drawn from consumption and investment. "The funds drawn from investment, are imputed a rate of return equal to their opportunity cost, which yields a shadow price" (Lyon, 1990:S-39). These future imputed capital costs, as well as future consumption benefits, are discounted back to the present at the MRS. If all benefits, B_t , are consumed, the present value of a public program using the shadow price of capital approach would be (Lyon, 1990:S-39)

$$\sum_{t=0}^T \frac{B_t - [(1-c)S + c]I_t}{(1+i)^t} \quad (1)$$

where

t = time periods ($0 \leq t \leq T$)
 B = benefits
 c = share of costs drawn from consumption
 (1 - c) = share of costs drawn from investment
 S = shadow price
 I = value of costs
 i = MRS

Many studies, including those by Bradford, Lind, and Mendelsohn, have shown the shadow price of capital approach is quite sensitive to necessary assumptions regarding the following parameters: a) reinvestment of capital; b) OCC rate; and c) MRS (Lyon, 1990:S-39-S-40). Considering the difficulty surrounding the empirical estimation of these

parameters, the shadow price of capital approach is not an instant solution to the MRT-MRS debate.

Government Involvement. During January, July, and August 1968, the Subcommittee on Economy in Government of the Joint Economic Committee, Congress of the United States, held hearings focused on the procedures used by agencies of the Federal Government in evaluating their investment programs. The hearings were especially concerned with the inconsistency in discounting procedures used among agencies and the existence of inappropriate methods in others. The subcommittee's findings were published in September, 1968, in the report titled "Economic Analysis of Public Investment Decisions: Interest Rate Policy and Discounting Analysis." (Subcommittee, 1968:III)

The subcommittee recommended,

no public investment be deemed 'economic' or 'efficient' if it fails to yield overall benefits which are as great as those which the same resources would have produced if left in the private sector. (Subcommittee, 1968:1)

Such a statement reveals their bias towards the OCC approach. The subcommittee was reluctant to endorse a purely OCC strategy because the government draws funds from both private consumption and investment. In continued support of its OCC related point of view, the subcommittee refuted alternatives suggesting the use of the MRS or the cost to the treasury of borrowing. (Subcommittee, 1968:10+)

As stated earlier, use of the MRS results in a lower SDR. This low rate would represent the collective desire of

society to transfer more income from current to future generations. Existence of such a collective goal is questionable. Even if it does exist, increasing the rate of economic growth would be a more successful means of realizing such a desire. A low, MRS-based SDR would actually impede rapid economic growth. Application of the cost-of-borrowing approach presumes the Federal Government maximizes revenues with respect to outlays, as does a private investor. Public decision-makers, according to the subcommittee's report, should maximize the difference between social benefits and the costs rather than net worth. (Subcommittee, 1968:10-15)

As a whole, the subcommittee accepted

without qualification the proposition that consistent discounting procedures and appropriate interest rate policy must be adopted throughout the Federal Government if wise and economic investment decisions are to be made. (Subcommittee, 1968:1)

They charged the Bureau of the Budget, in conjunction with the appropriate agency, to undertake a study to develop a method of numerically estimating a weighted-average SDR associated with government spending. It must be noted, however, several members, including Representative Patman, disagreed with the conclusion that public investments can be evaluated in a manner similar to private investments. He stated,

Reliance on profit oriented business criteria to evaluate government investments would inevitably result in the abandonment of projects with more potential and far-reaching benefits . . . in favor

of those which showed an immediate financial return. I believe this would be disastrous to the fulfillment of a whole range of the goals of our society. (Subcommittee, 1968:22)

As in the realm of economics, politics has also been unable to resolve the disparity between opposing views regarding the selection of a SDR. Unlike the theoretical nature of economics, the political environment demands a choice.

Policy Guidance. OMB Circular No. A-94, March 27, 1972,

prescribed a standard discount rate to be used in evaluating the measurable costs and/or benefits of programs or projects (of the Federal Government) when they are distributed over time. (Office of Management, 1972:1)

This edition rescinded the June 26, 1969 publication. The following list is only an extract of the many procedures established by the circular: a) all economic analyses will be evaluated in constant dollars with considerations for changes in relative prices; b) variability in predictions should be evaluated via sensitivity analysis; and c) programs and projects subject to the guidance of the circular shall use a discount rate of 10 percent. This rate represents an average rate of return on private investments before taxes and after inflation. (Office of Management, 1972:2-4)

DoD Instruction 7041.3, December 19, 1966, Economic Analysis and Program Evaluation for Resource Management, outlined policy guidance and established a framework for the consistent application of economic analysis to DoD proposed

programs, projects, and activities and program evaluations of on-going activities. This edition reissued the February 26, 1969 publication. DoD Instruction 7041.3 clearly supports the OCC approach in selecting a SDR:

This policy is based on the premise that no public investment should be undertaken without explicitly considering the alternative use of the funds which it absorbs or displaces. This imposes an appropriate opportunity cost for the capital required to finance the alternative. . .
(Department of Defense, 1972:6-7)

As stated by the OMB Circular A-94, this Instruction mandates a 10 percent real DR and allows for the use of other rates for sensitivity analysis. (Department of Defense, 1972:1+)

United States Air Force Regulation 173-15, Economic Analysis and Program Evaluation for Resource Management "outlines the policies and procedures and assigns responsibility for preparing and evaluating an economic analysis or program analysis" (Department of the Air Force, 1988:1). It implements DoD Instruction 7041.3, 18 October 1972, and references the policy guidance of OMB Circular No. A-94. AFR 173-15 "is not intended to replace the judgment of the decision-maker, but rather to aid that judgment" (Department of the Air Force, 1988:3). When evaluating alternative allocations of resources, the regulation directs the use of a 10 percent real DR and a 5 percent real DR for sensitivity analysis. An upper limit to the sensitivity analysis is not specified.

Current Status. (Lyon, 1990:S-29-S-50) There are three federal oversight and budget agencies: a) the OMB; b) the Government Accounting Office (GAO); and c) the Congressional Budget Office (CBO).

In principle, the discounting policies of OMB, GAO, and CBO can effect virtually all federal investment, regulatory, lease-purchase, and asset-divestiture programs through their impacts on either executive or legislative decisions. (Lyon, 1990:S-30)

As of March 1990, all three were reviewing their DR policies.

OMB's DR policy guides the cost-benefit analyses of all executive branch agencies. Currently, it applies a 10 percent real DR except in the following cases: a) alternative DRs are applied if justifiable; b) lend-purchase decisions and water projects investments are based on Treasury borrowing costs; and c) asset-divestiture ventures are evaluated according to market interests rates for comparable private-sector endeavors. The 10 percent rate is consistent with Stockfish's (1969) estimated weighted average of returns to unregulated-corporate, public-utility, and noncorporate capital. Those reexamining OMB's DR policy have expressed interest in selecting their SDR by an approach based on the shadow price of capital.

GAO is a congressional agency, not an executive department, and therefore not bound by OMB policy. It does not distinguish between public investment, lend-purchase, or asset-divestiture analyses. GAO bases its SDR on the

average nominal yield of marketable Treasury debt. A real rate is obtained by subtracting forecasted inflation from the Treasury rate. Current review of its policies appears to favor retaining the Treasury borrowing rate approach with significant emphasis placed on sensitivity analysis.

CBO, though it is also a congressional agency, has established its own DR policy. CBO's SDR, estimated to be 2 percent, is based on the real yield of Treasury debt. In the case of valuing assets such as loans made by the Federal Government, they recommend using comparable private-sector interest rates. Reassessment of CBO's procedures indicates a bias towards retaining established policies with significant emphasis placed on the use of sensitivity analysis.

In 1968, Baumol candidly surmised the disparities surrounding the debate among public economists over the selection of a SDR. Twenty years later, Lyon submits:

Specialists now better understand weighted discount rates, shadow prices, and the effects of risk. On the other hand, the range of possible shadow prices alone suggests that the range of analytical outcomes may even be wider and more sensitive to assumptions than realized years ago. (Lyon, 1990:S-46)

Précis. The intent of the SDR section of this literature review is to provide the reader with an appreciation for the complexity of the debates surrounding this issue and a basic understanding of the different approaches endorsed by respected economists. It is also provides an important introduction to the directives and

regulations mandating the use of discounting in DoD acquisition proposals. To transition into the next phase of the discussion linking CRRs with the SCC, the reader must assume the perspective of those proponents advocating the OCC approach to determining the SDR. The next section will review the use of IRRs in capital budgeting.

Capital Budgeting and the Internal Rate of Return

Government. (Seagraves, 1970:430+) Seagraves proposed an operational framework which he hoped would help bring about some agreement on the issues involved in selecting projects for public investment. He advocated,

the complete separation of the calculation of real internal rates of return for projects, which should be fairly stable over time, from calculation of the real social opportunity cost of capital, which may fluctuate for cyclical and other reasons. (Seagraves, 1970:430-431)

Seagraves suggested the government should chose those projects with the highest IRRs while first staying above the SDR. His main pretense for using IRRs, instead of NPVs or cost-benefit ratios (CBRs), to rank competing projects "is simplicity in that it does not assume prior knowledge of the social (discount) rate or have to be recalculated as (this) rate changes" (Seagraves, 1970:436).

Proponents of the use of NPVs and CBRs challenge the use of IRRs for two fundamental reasons. The main argument with the determination of IRRs is the potential existence of multiple roots in a NPV relationship; this implies the existence of multiple IRRs. Application of the IRR approach

is also limited by assumptions concerning the reinvestment of cash inflows (see Appendix A). If reinvestment is not on equally favorable terms as the IRR, the use of IRR is not neutral to the time profile of the cash flows.

Seagraves responded to each of these criticisms. In the case of multiple IRRs, he advocated selecting the highest rate characterized by both a $NPV = 0$ and a negative slope. Point D, in Figure 1, satisfies these criteria while Point E does not meet the negative slope criterion. Cannaday added a third condition to the determination of relevancy when dealing with multiple roots to the NPV relationship. He concluded the IRR must be greater than -1 to be considered relevant (Cannaday, 1986:32). The reinvestment restraint is not as easily reconciled.

Seagraves acknowledged, when a given amount of capital must be allocated to a variety of current investments, as is typical in government investment, "the present value criterion lends itself better . . . to complete investment of the available capital" (Seagraves, 1970:438). This admission becomes relevant when the following is considered:

The rate of interest used in (NPV) discounting should be the same as the internal rate of return of the marginal project, and this assumes reinvestment of cash flows at returns equal to those of the least acceptable project.
(Seagraves, 1970:438)

The necessity of the reinvestment assumption is not unique to the IRR approach and can not be isolated as the reason to discredit Seagraves' methodology. Additionally, when

limited by the marginal-project constraints, the NPV calculations necessary for project ranking must be done using iterative integer programming. Seagraves claims the rankings resulting from these cumbersome calculations "would be practically the same (as those) using internal rates of return and settling for approximate exhaustion of the capital available" (Seagraves, 1970:438). Advancements in economic theory and computational tools have proven this accusation to be wrong (Christensen, 1991).

Seagraves denotes two common concerns related to the ranking of projects by their NPVs or CBRs. When NPV is used, the choice depends on the size of the project, the DR embraced, and the project's economic life. When CBR is used, the choice depends on the DR and the time profile of the project. Rankings by NPV are especially susceptible to bias due to project size. If relatively lower DRs are used, the resultant rankings from either approach will be biased in favor of the projects with longer lives.

In contrast to NPVs and CBRs, the IRR does not depend on an interest rate being selected and generally provides a reasonable basis for comparing projects with widely different time horizons. In other words, projects can be ranked by their IRR even though the SDR has not been selected. When the SDR is identified, the rankings only require comparison, not recalculation. The ranking of projects according to their IRRs remains isolated from real interest rates because

real internal rates of return stand as a summary of the "technological efficiency" of a project as long as relative prices stay the same. (Seagraves, 1970:439).

Evaluating the Use of Internal Rates of Return. Since capital budgeting decisions take place in an uncertain environment, the use of any "theoretical rationing model must be supplemented with sensitivity analysis to be useful" (Hsiao and Smith, 1978:645). Hsiao and Smith developed an analytical approach which provides guidelines to the decision-maker using the IRR model for situations in which forecast errors are likely. They defined sensitivity as the first partial derivatives of the CBR relationship, Eq (2), with respect to r (Hsiao and Smith, 1978:645):

$$c \left[\frac{1}{r-a} \right] \left[1 - \left(\frac{1+a}{1+r} \right)^T \right] = 1 \quad (2)$$

where

c = cash-flow coefficient, ratio of first period's cash flow to the original investment
 r = IRR
 a = growth rate of net return stream, constant
 T = economic life of the project

Table 3 lists the results from differentiating Eq (2) with respect to each of the parameters while holding the other two constant, that is, assuming they are correctly estimated. The positive correlation between r and the input parameters indicates an overestimation (underestimation) of any parameter will result in an overestimation (underestimation) of the IRR.

TABLE 3

Effects (Increase (+) or Decrease (-)) of Increases in
T, a, and c on Time (TS), Change-Rate (CHS), and
Cash (CS) Sensitivities (Hsiao and Smith, 1978:647)

	<u>r</u>	<u>TS</u>	<u>CHS</u>	<u>CS</u>
T	+	-	+	-
a	+	+	-	+
c	+	-	+	-

Note: CHS and CS refer to the parameters a and c,
respectively.

The following example explains the three remaining columns of the table. As projects with larger constant growth rates are evaluated, the time and cash sensitivities of the IRR model become more sensitive to variations in the predicted value of a; the change-rate sensitivity of the IRR model decreases for larger values of the constant. Though no sub-class of projects is insensitive to errors, Hsiao and Smith submit, "projects with long lives, large cash-flow coefficients, and quickly declining cash flows are among the least sensitive of projects" (Hsiao and Smith, 1978:648).

Moore and Chen proposed a statistical approach for dealing with the parameter uncertainty of expected cash-flow streams. They suggest selecting or rejecting a project based upon the probability that its IRR will be less than the organization's required rate of return. Application of their method requires the availability of sample data from similar investments previously undertaken.

By means of a Bayesian approach, this data is used to derive a predictive distribution for the project. The distribution is used to simulate cash flows which are then solved for IRRs. Finally, the statistical qualities of the dispersion of these IRRs are used to assess the probability introduced above. Moore and Chen warn that ignoring parameter uncertainties leads to an understatement of the total riskiness of an investment proposal and offer their approach as a more conservative tool to the analyst required to make an accept/reject decision. (Moore and Chen, 1984:351+)

Capital Budgeting Practices. Contemporary literature reveals a "capital budgeting revolution for the 1990s." Companies are becoming increasingly more aware of the crucial importance of investment decisions to their long-term success or failure. Many studies, primarily based on surveys, indicate a particular shift in emphasis towards investment selection techniques and post-completion audit procedures. Of the firms surveyed which report using formalized capital budgeting techniques, most rely on one of the following DCF techniques: IRR, NPV, payback period. There appears to be no consensus among the many studies to which of the three techniques is most widely accepted. (Pike, 1988:28+)

Several of the studies report large corporations, such as Fortune 500 members, favor the IRR approach (Blazouske, 1988; McIntosh, 1987; Moore, 1989; Mukherjee, 1988;

Schmidgall, 1990). A survey of the major lodging chains revealed 74 percent of those respondents using capital budgeting techniques based investment decisions on IRR analyses. Comparison of the these results to a previous study of nonhospitality firms showed the lodging industry to be lagging behind the service industry as a whole in the use of IRRs. (Schmidgall, 1990:40+)

Several surveys of other industries and firms concur with Schmidgall's findings: a) using published capital budgeting manuals as his data source, Mukherjee found most manuals direct the use of DCF tools for analysis, and the IRR is the most popular choice (Mukherjee, 1988:28+); b) among the 41.6 percent chief financial officers of corporations listed by Canada's 1985 Financial Post 500 Industrials, IRR was the most commonly used method for evaluating investment projects (Blazouske, 1988:51+); c) 79 percent of the 37 responding corporate real estate executives said they use IRR in their investment analyses (McIntosh, 1987:125+); and d) a logistic regression of a multivariate model, based on the responses from 313 of the 1989 Fortune 500 companies, predicted a link between three analytical techniques (financial leverage, inventory management, and IRR) and positive firm performance (Moore, 1989:79+).

Advocates of the IRR approach are not without opponents. The main argument against its use in the

corporate sector deals with the reinvestment rate issue. The IRR assumptions hold that the remaining devoted capital and the cash recovered from an investment opportunity grow at the same IRR originally offered by the venture. Analysis has shown the growth rate of cash recoveries to be the firm's marginal growth rate (MGR) and not the IRR. Only in the unlikely case of a MGR equal to the IRR will the conventional reinvestment rate assumptions be correct (Lohman, 1988:303+).

There have been several modifications made to the general definition of the IRR in response to the reinvestment rate criticisms (Beaves, 1988; Hartley, 1988; Sweeney, 1987). One model handles non-IRR reinvestment rates by basing the fundamental DCF calculations on separate firm-borrowing and -lending rates. This approach is justified by proving it does not jeopardize the wealth maximization objective inherent in NPV analyses (Sweeny, 1987:19+). In addition to wealth maximization, independent reinvestment rates are credited with two other valuable contributions to capital budgeting: a) the intrinsic capability of second order sensitivity analysis; and b) elimination of multiple roots in NPV solutions for the majority of situations (Beaves, 1988:275+). Despite the many denunciations, use of the IRR approach to capital budgeting in the corporate sector is becoming more and more widespread.

The Department of Defense's Potential Use of the Internal Rate of Return. (Christensen, 1991) Recall the OCC orientation urged at the end of the Social Discount Rate section of this literature review. If the SCC is to truly measure the opportunities lost in the private sector by government spending, a national-level IRR may provide a unique understanding of the impact of public investments on the economy. The derivation of a nationally representative IRR would offer the opportunity to update and possibly improve the 10 percent SDR mandated by the OMB.

The results of such an undertaking may provide an DR alternative considered too large to serve as the SDR. Since corporations must consider their long term welfare, they will usually invest in all capital ventures providing a return greater than their estimated COC. Based on this assumption, the average of the IRRs of a firm's active projects would be greater than its COC. Extrapolation of this illustration to the national level would plausibly result in the overestimation of the true SCC.

Such an outcome would not annul the practicality of the endeavor. AFR 173-15 directs the use of a 10 percent DR for cost/benefit analysis as well as a 5 percent DR for sensitivity analysis. No upper limit to the sensitivity analysis is required. A DR which overestimates, but accurately represents a national level IRR, would be a reasonable upper limit to a sensitivity analysis aimed at accurately assessing the opportunity foregone by private

investing as a result of additional DoD spending. The crux of this discussion hinges on the ability to measure firm profitability via currently unobtainable IRRs. CRR theory and its resulting IRR-estimation models offer a solution to this problem.

Précis. Hopefully, the Capital Budgeting and Internal Rate of Return section has reinforced the reader's awareness of the need for discounting in capital budgeting, provided examples of the IRR's value in analyzing investment propositions, and introduced the IRR's potential as a tool to augment the established practices of evaluating the SCC. It is difficult to calculate firms' profitability based on published financial information. Developed as an outgrowth of Ijiri's reemphasis on cash-flow accounting, the CRR offered a new instrument to estimate the elusive IRR.

Cash Recovery Rate

Basic Theory.

Foundation. (Ijiri, 1978:331-333) "Needless to say, cash flow is the basic objective in business." Yuji Ijiri, in the summer of 1978, proposed reexamination of the inherent values of cash-flow oriented accounting in response to the Financial Accounting and Standards Board's (FASB) Discussion Memorandum entitled "Elements of Financial Statements and Their Measurement." The memorandum was part of the FASB's "Conceptual Framework for Financial Accounting and Reporting" project, an endeavor to develop a unified

conceptual framework for financial accounting and reporting.

The American Institute of Certified Public Accountants

(AICPA) stated in a report published in 1973:

An objective of financial statements is to provide information useful to investors and creditors for predicting, comparing, and evaluating potential cash flow to them in terms of amount, timing, and related uncertainty.

Ijiri, building on this statement by the AICPA, suggested that well-classified data on past cash flows of a firm were more directly relevant to these objectives than data on assets and liabilities. Many economists from this era contended with the dichotomous practice of basing investment decisions on forecasted cash flows and then evaluating performance on the grounds of profit. William Ferrara, in his December 1976 article titled "Accounting for Performance Evaluation and Decision-making," asserted,

It is impossible to argue against the applicability of discounted cash-flow concepts for decision-making. Thus, there really is no choice but to convert performance evaluation from accrual accounting to cash-flow accounting. Otherwise, we will put managers in the intolerable position of making decisions one way and having their performance evaluated another way.

Unfortunately, the negative attitudes towards cash flows held by the accounting profession, including the Accounting Principles Board, largely eliminated the growing use of cash-flow figures in annual reports experienced in the 1950's. Ijiri hoped to inject new spirit into once vibrant cash-flow accounting by introducing modern concepts in capital-budgeting decisions and financial reporting.

Development. CRR theory assumes cash flows are central to performance evaluation and accounting practices should focus on their use. Eq (3) is the fundamental cash-flow accounting relationship (Ijiri, 1978:335):

$$\text{Investment-Recovery} = \text{Financing-Repayment} \quad (3)$$

Classifications for all firm cash flows governed by Eq (3) are displayed in Figure 3. Investment cash flows are used

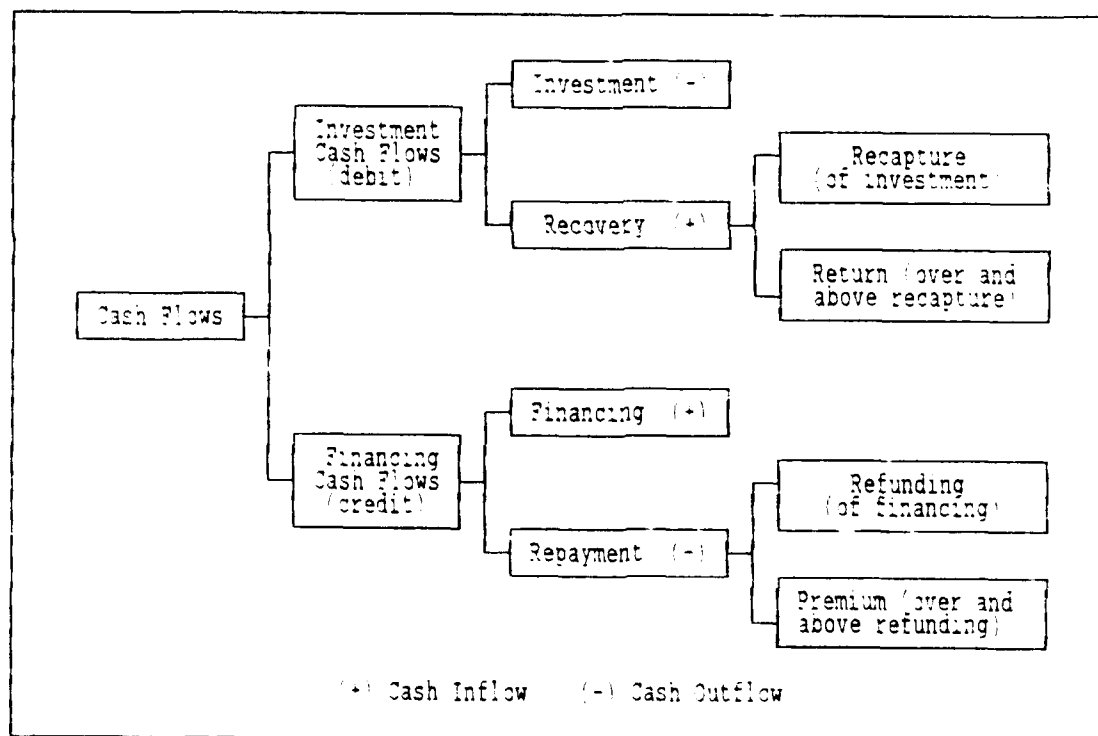


Figure 3. Classification of Cash Flows (Ijiri, 1978:335)

to calculate the recovery rate and the financing cash flows are used in repayment rate calculations; repayment rate will

be ignored by this research. Eq (4) displays the basic definition of the recovery rate (Ijiri, 1978:338):

$$\text{Recovery Rate} = \frac{\text{Recoveries}}{\text{Gross Investments}} \quad (4)$$

Unfortunately, cash-flow information was not, and is not, readily ascertainable from corporate-environment financial statements. Considering these limitations, Ijiri developed Eq (5) to provide a financial statement-derived approximation of the recovery rate (Ijiri, 1980:55):

$$\text{Cash Recovery Rate} = \frac{\text{Cash Recoveries}}{\text{Gross Assets}} \quad (5)$$

where

Cash Recoveries = (funds from operations)
+ (proceeds from disposal of long-term assets)
+ (decrease in total current assets)
+ (interest expense)

and

Gross Assets = (total assets) + (accumulated depreciation), averaged between beginning and ending balances.

"Gross assets do not coincide with gross investments, due to such factors as investments that were expensed, idle assets, and surplus cash" (Ijiri, 1980:55). Under many cases, especially on a corporate scale, gross assets provide a reasonable substitution for gross investment. Periodic cash flows are well represented by the factors included in the numerator of Eq (5). "Thus. (we) may calculate corporate

recovery rate based on the relationship in (Eq (5))" (Ijiri, 1980:55-56). Ijiri calculated the recovery rate for 20 corporations using this equation and the data from published financial statements. The CRRs displayed stability, considered a desirable characteristic, throughout the seven-year period analyzed. Table 4 lists the CRRs for six of the companies; Appendix B lists the information for all 20 companies.

TABLE 4
CASH RECOVERY RATES (Ijiri, 1979:261)

	Cash recovery rate					
	1972	1973	1974	1975	1976	Avg.
IBM	18.2	19.0	18.8	18.3	18.7	18.6
General Motors	14.6	15.2	9.9	12.0	16.1	13.6
Gulf Oil	10.7	13.3	12.9	10.5	11.5	11.8
Goodyear	8.4	8.1	7.8	7.1	7.3	7.7
Westinghouse El	7.7	8.1	6.4	6.7	6.9	7.2
U.S. Steel	4.3	6.3	9.5	7.4	6.4	6.8

Coupled with data on the economic life of a project and assumptions about its cash-flow patterns, an estimate of the project or corporate DCF rate (see Appendix A) may be developed from its annual CRRs.

Convergence of the Cash Recovery Rate. Central to the proposed use of the CRR as a surrogate for the IRR is Ijiri's demonstration that under general conditions the

recovery rate converges to a constant, R, which is equal to the capital recovery factor (CRF) (Ijiri, 1979:259):

$$R = \frac{i}{[1 - (1+i)^{-n}]} \quad (6)$$

where

i = DCF rate

n = economic life (see Appendix A)

Table 5 shows the CRRs of earlier years fluctuate widely around the CRF, R = .6545, but move closer to this constant value in the later years. Stauffer also demonstrated the

TABLE 5
RECOVERIES AND REINVESTMENTS (Ijiri, 1979:260)

	Recoveries and (reinvestments)						
	0	1	2	3	4	5	6
Year 0	(1)	0.6	0.72				
1		(0.6)	0.36	0.432			
2			(1.08)	0.648	0.7776		
3				(1.080)	0.6480	0.7776	
4					(1.4256)	0.8554	1.0264
5						(1.6330)	0.9798
6							(2.0062)
Recoveries		0.6	1.08	1.080	1.4256	1.6330	2.0062
Investments		1.0	1.60	1.680	2.1600	2.5056	3.0586
Recovery rate		0.6	0.675	0.643	0.6600	0.6517	0.6559

firm's CRR converged to a constant when all cash flows were reinvested (Stauffer, 1971: 451).

(The CRR is) an excellent approximation of the DCF rate if the recovery rate is over 15 percent and

the project life is over 15 years, which is perhaps the case in a majority of capital investment decisions. (Ijiri, 1980:55)

Implicit to the CRF calculations are the assumptions of total reinvestment of recoveries and constant cash-flow profile. When cash flows are level (constant), the firm's investment growth rate has no influence on the CRR (Salamon, 1985:499).

Corporations invest in projects with a variety of cash-flow patterns, recovery rates, and economic lives. If it is reasonable to assume the mix of projects and their econometric characteristics are reasonably stable over time, then corporate investments may be regarded as repeated investments in a composite project with a given cash-flow pattern and economic life (Ijiri, 1979:260-261).

For a mature corporation, this may be a reasonable assumption. While it is true that a corporation continually seeks new ventures, their impact on the composite project seems to be relatively small in many cases. Under the assumption that a mature corporation reinvests its recoveries in each year in the same composite project, the convergence theorem assures that in the long run the corporate recovery rate converges to the capital recovery factor almost regardless of the cash-flow pattern of the composite project. (Ijiri, 1979:261)

Ijiri, 1979, provided a mathematical proof for the convergence of recovery rates. Within this proof, he converted the cash recovery pattern from a nominal basis to a real basis. This is done to allow the present derivation to build upon his 1967 proof of the convergence of annual investments (Ijiri, 1967:321+). Both analyses combine to support the postulate that the CRR converges to the CRF for

any cash recovery pattern, providing it is not cyclical in nature (Ijiri, 1979:267).

The economic significance of the CRR's convergence extends from the implication that by estimating the economic life of the composite project, n , the DCF rate, i , can be derived from the CRF, R , by solving Eq (6). Ijiri asserts R may be approximated by an average CRR, see Table 4 and Appendix B for examples, in order to solve Eq (6) for i . This approach forms the foundation for the theory which suggests the CRR functions as a surrogate for the IRR.

Capital Budgeting. (Ijiri, 1978:342-347)

Appendix A provides definitions for all of the cash flows discussed in this subsection. The cash-flow concept used in capital budgeting generally deals with proper cash flows. This perspective must expand to include two additional cash-flow categories: constructive cash flows and residual cash flows. As an example of constructive cash flows, consider the exchange of land, in lieu of cash payment, for a firm's stock. It is better to assume there occurred both a financing and investment activity (see Appendix A) than to consider there was no cash flow of any kind.

"The use of residual cash flows becomes important in dealing with the turnover of assets and liabilities that are of short-term nature." Residual treatment maintains recoveries are only recognized when current assets are decreased in total and investments are only recognized when total current assets increase. If \$100 worth of merchandise

was purchased and \$75 was sold during the year, the residual perspective ignores the initial investment and partial recovery and simply acknowledges the \$25 investment cash flow. By encompassing constructive and residual cash flows, Ijiri's cash-flow accounting propositions are distinguished from the traditional cash basis of accounting and are brought closer in line with the cash-flow concept used in capital budgeting.

Corporate Profitability. As discussed previously, there exists a dichotomy between the methods used to select among capital investment proposals and the manner by which the performance of existing projects is evaluated. Return on investment (ROI) is a more advanced concept than CRR, since ROI includes considerations for depreciation and other noncash items. "The question is whether such additional adjustments make the (performance) indicator more useful for management" (Ijiri, 1980:58).

Performance measurements based on earnings are subject to many discretionary judgments of the corporation. Economic life is perhaps one of the most uncertain factors in investment decisions. Ijiri (1980) suggested it would be best to leave it out of the indicator rather than incorporating it in based on an accountant's judgment. Noncash items, which are relatively ambiguous, are often mixed with cash items, which are objective. "Like adding miles to inches, such a mixture (reduces) the reliability of the resulting (indicator)" (Ijiri, 1980:55).

"The recovery-rate approach is, on the other hand, strictly based on cash flows which are more objective and less subject to arbitrary discretion of the (firm)" (Ijiri, 1979:261). Application of the CRR ignores economic-life estimates until the very end of the profitability determination. This provides the user of the financial data with flexibility to handle uncertainties in a manner best suited to their needs. True performance of a project cannot be properly evaluated until it is terminated. Ijiri's cash-flow accounting and CRR contributions present methods to deal with uncertainty in future cash flows in order to make an assessment of current projects before the end of their economic lives (Ijiri, 1980:60). Future cash-flow patterns

may be different from those indicated by the past cash-flow data. . . the purpose of the cash-flow statement is not to present such projections, but to provide information useful for predicting, comparing, and evaluating potential cash flows. Therefore, recovery and repayment rates in the past are presented as a useful means by which to make such a prediction, comparison, and evaluation of potential cash flow. (Ijiri, 1978:341)

CRR to DCF Rate. By building on his proof that, under certain conditions, annual reinvestments converge to a constant, Ijiri demonstrated similar behavior for the cash recovery rate. When all necessary cash-flow data are known a priori, the CRF can be used to calculate the theoretical value upon which a firm's annual CRRs will converge. Unfortunately, the DCF rate, used in the CRF equation, is not known before hand. Ijiri substitutes an average of annual CRRs for the indeterminate value to which these

annual rates converge. The CRR-CRF relationship is then applied posteriori to derive the DCF rate (see Appendix C).

Advancements. (Salamon, 1982:292-302) Gerald L. Salamon, concerned about the hazards of using the accounting rate of return (ARR) (see Appendix A) to measure firm profitability, continued development of the CRR theory formulated by Ijiri. His first addition was to replace Ijiri's use of the term DCF rate, i , with IRR, r . Salamon recommends firm profitability and ranking be based upon IRRs estimated from CRRs. Salamon pointed out most firms do not reinvest all of their cash recoveries. His economic performance model considers the link between the CRR and the IRR when a firm does not reinvest all of its cash flow as well as the impact of inflation on this link.

It is important to note that within (Salamon's) model there is still a link between the firm's cash recovery rate and its IRR which is largely independent of accounting method and can be utilized to obtain empirical estimates of the firm's IRR. (Salamon, 1982:293)

This fact is emphasized by Salamon's re-estimating the IRRs for the 20 firms previously evaluated by Ijiri (1980) (see Appendix B for Ijiri and Appendix D for Salamon).

The model developed by Salamon (1982) has the same structure as the model he used to examine the relationship between the firm's IRR and the IRR of its projects (Salamon, 1973:298-300). Both assume the firm is a collection of projects that have the same useful life, cash-flow pattern, and IRR. It also assumes the firm has a constant rate of

growth in real growth investment. Therefore, a project acquired in any year is different from those of other years only in scale and the differential is a function of the firm's growth rate. The final constraint placed on the CRR-IRR model assumes the firm operates in an environment in which there is a constant rate of change in the level of all prices and all prices change according to this one rate.

The firm's CRR, for any year $n + j$ ($j \geq 0$), converges to the constant ρ in Eq (7) (Salamon, 1982:297):

$$\rho = \left[\frac{(1-pg)p^n g^n}{1-p^n g^n} \right] \left[\frac{g^n - b^n}{g^n(g-b)} \right] \left[\frac{r^n(r-b)}{r^n - b^n} \right] \quad (7)$$

where

$p = 1 + p'$
 $g = 1 + g'$
 $r = 1 + r'$
 n = economic life of all projects
 b = cash-flow parameter

and

p' = annual inflation rate ($p' > -1$)
 g' = annual growth rate in real gross investment
 ($g' > -1$)
 r' = real IRR of all firm projects ($r' > -1$)

The three separate terms within square brackets on the right-hand side of Eq (7) indicate that a firm's cash recovery rate is a function of (1) the relation between the inflation rate and the real growth rate in gross investment, (2) the relation between the growth rate in gross investment and the cash-flow parameter, and (3) the relation between the cash-flow parameter of firm projects and the IRR of the firm. Additionally, the cash recovery rate is dependent on the useful life of firm projects. (Salamon, 1982:297)

Eq (7) is applicable only to a firm that has existed for at least as long as the economic life of its composite project.

The parameters p' and g' , for a period that is reasonably close to n , were estimated as follows (Salamon, 1982:298):

$$\hat{p}' = \frac{1}{19} \ln \left[\frac{CPI_{78}}{CPI_{59}} \right] \quad (8)$$

$$\hat{g}' = \frac{1}{19} \ln \left[\frac{\frac{GA_{78}}{CPI_{78}}}{\frac{GA_{59}}{CPI_{59}}} \right] \quad (9)$$

where

\hat{p}' = estimator of p'

\hat{g}' = estimator of g'

CPI_{xx} = the consumer price index on 12/31/xx

GA_{xx} = the gross assets of the firm on 12/31/xx

The cash-flow parameter, b , is not readily ascertained.

Firms publicize very little information on their "typical" b . In order to determine how dependent the IRR calculation is on the estimated parameter b , Salamon examined each firm using two profiles, .8 and 1.

The fact that the cash-flow pattern is assigned a value (and is not estimated) means that the estimates of IRR that result from solving Eq (7) for r have to be viewed as conditional rather than as unconditional estimates. (Salamon, 1985:498)

He acknowledged this tactic only scratches the surface of the cash-flow parameter question since different industries, and even different firms within an industry, would invest in projects with different typical cash-flow parameters.

In his 1985 "Accounting Rates of Return" article, Salamon expanded the study of project ranking according to

conditional IRRs' and the ranking's insensitivities to the value assigned to b . He calculated four IRRs for each firm using cash-flow parameter values of .8, 1.0, 1.1, and a random quantity selected from a uniform distribution defined by the interval (.8, 1.1). The randomly selected value allowed for analyses in an environment where all firms were not assumed to share a common cash-inflow profile. The conditional IRRs shown in Table 6 seem to behave somewhat independently of the assumed cash-flow parameters.

TABLE 6
ESTIMATED INTERNAL RATES OF RETURN (Salamon, 1982:297)

Firm	n	q	Avg CRF	n = 20		n = 5		IRR(4)	IRR(5)	IRR(6)
				b = 1		b = .8				
				IRR(1)	IRR(2)	IRR(3)	IRR(4)			
IBM	3	3.5	12.3	12.3	12.3	12.3	12.3	12.3	12.3	12.3
General Motors	3	3.5	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0
Gulf Oil	3	3.5	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0
Goodyear	3	3.5	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0
Westinghouse E.I.	3	3.5	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0
U.S. Steel	3	3.5	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0

Ijiri's use of $n = 20$ years in his CRF calculations lead to over-estimating the IRR for firms with useful lives less than 20 years and underestimating the IRR for firms with useful lives greater than 20 years. To estimate the useful life of each firm's projects, \hat{n} , Salamon divided the average of the firm's gross plant for the year by its depreciation expense for that year. He did this for each

year between 1972 and 1978 and then averaged the seven values to determine \hat{n} of the each firm's typical project.

Table 6 compares IRRs calculated from average CRRs using Ijiri's Eq (6), IRR(1,4), and Salamon's Eq (7), IRR(2,3,5,6) (See Appendix D for a listing of all 20 firms). Because Eq (6) calculates a nominal rate and Eq (7) a real rate, it is not surprising Salamon's estimated IRRs are uniformly lower than Ijiri's given the inflation rate for the period under observation.

"The levels of the profitability measures depicted in (Table 6) are not nearly as important as is their degree of similarity or dissimilarity" (Salamon, 1982:299). Table 7 suggests the IRR estimates are most sensitive to the project-life assumption. The IRRs based on a constant life

TABLE 7

PEARSON CORRELATION COEFFICIENTS (Salamon, 1982:300)

	IRR(1)	IRR(2)	IRR(3)	IRR(4)	IRR(5)	IRR(6)
IRR(1)	1.000					
IRR(2)	.998	1.000				
IRR(3)	.968	.949	1.000			
IRR(4)	.717	.695	.776	1.000		
IRR(5)	.741	.725	.767	.996	1.000	
IRR(6)	.583	.543	.708	.942	.907	1.000

Note: Input parameters for IRR(1-6) are as follows: IRR(1) (from Eq (2)), $n = 20$; IRR(2) $n = 20$, $b = 1$; IRR(3) $n = 20$, $b = .8$; IRR(4) (from Eq (4)) $n = \hat{n}$; IRR(5) $n = \hat{n}$, $b = 1$; and IRR(6) $n = \hat{n}$, $b = .8$.

of 20 years, $IRR(1,2,3)$, are highly correlated, as are the IRRs based on varying lives of \hat{n} , $IRR(4,5,6)$. The low correlation between associated IRR pairs that differ in the use of n , $IRR(1 \text{ and } 4, 2 \text{ and } 5, 3 \text{ and } 6)$, reinforces the earlier statement that firms are most likely composed of projects with different composite lives. An examination of Table 7 for

pairs of IRRs that differ only as to the level of the cash-flow pattern's parameter ($IRR(2 \text{ and } 3, 5 \text{ and } 6)$) suggests that a ranking of a set of firms by their IRRs would be largely unaffected by whether the IRR was calculated assuming level cash flows or declining cash flows. (Salamon, 1982:301)

Contrary to the above stated insensitivity, the average level of the conditional IRR displays an obvious dependence on the level of the parameter b . In a sample of 965 industrial firms, it was noted the average estimated IRR (.1229) based on $b = .8$ is greater than the average estimated IRR (.1079) based on $b = 1$. (Salamon, 1988:278)

Salamon's modeling of the CRR-IRR relationship, Eq (7), incorporates Ijiri's assumption that the project mix of a mature firm can be characterized as a composite project with a given life and cash-flow parameter. Salamon began with the CRF, Eq (6), and added considerations for the firm that does not reinvest all of its recoveries and must operate in an inflationary environment. He demonstrated estimated IRRs are very sensitive to the firm's assumed project life. Salamon's work has made it

clear that (parties interested in a theoretically defensible measure of firm profitability) are

closer than ever before to having an empirical measure of the profit performances of firms (CRR) which is directly linked to a discounted cash-flow rate of return (IRR). (Salamon, 1982: 302)

Challenges and Limitations. ". . . Salamon's optimistic conclusion (previous quote, Salamon, 1982:302) is too sanguine" (Brief, 1985:473). Brief categorized the CRR method's set of assumptions so restrictive that it limits practical usefulness of the CRR. Salamon developed Eq (7) assuming cash flows grow indefinitely at a constant rate. Since this specifies a firm's future cash flows and since historical cash flows can be calculated, "the CRR in effect, assumes that the firm's entire stream of cash flows is known" (Brief, 1985:474). The IRR could then be determined directly from the firm's cash flows and there would be no need to estimate a composite project life, n , or select a cash-flow parameter, b .

Some researchers have questioned whether information about a firm's past and future cash flows even exists. Others suggest such estimates are far too subjective to be of any practical use.

Since the CRR method was devised as a general method of estimating a firm's IRR, its usefulness depends on whether or not the environment reflects these (future cash flow) assumptions thereby giving the predicting model in (Eq (7)) external validity. The issue of external validity has not been addressed. Until it is, the CRR method has no justification. (Brief, 1985:474)

The growth of firms is variable and difficult to predict. Empirical work in this area, even if undertaken, is not likely to prove the assumptions underlying the CRR

correspond to the real world and therefore, "the potential of the CRR method is in doubt" (Brief, 1985:474).

Stark (1987b) identified a divergence between the definition given to the CRR derived by mathematical model-building and the CRR calculated from Ijiri's empirical definition (Eq (5)). He labeled the CRR implied by the model-building exercises, Ijiri (1978, 1979) and Salamon (1982, 1985), the true cash recovery rate (TCRR) and the CRR used empirically as the empirical cash recovery rate (ECRR). Stark's reproach alleges the TCRR is unobservable directly from published financial statement data if the firm has current asset balances and the ECRR does not behave as a proxy for the TCRR.

Consider a firm that conforms to all of Ijiri's and Salamon's assumptions necessary to characterize it as a composite project. If this firm is constrained to operate in a no-growth situation, the sum of the reduction in current assets during the year n associated with each project will be equal to the initial current asset investment for the firm's typical project. Thus, the balance in current assets does not change. To determine the numerator of the TCRR, the recoveries component of the current asset balance must be separated from the investment component. Current format of financial-statement information impedes such a distinction and therefore renders the TCRR unobservable. (Stark, 1987b:99-101)

Continuing with the firm identified in the previous paragraph, Stark reveals the ECRR will systematically underestimate the TCRR by an amount equal to the recoveries of the current assets (see formulation in Appendix E). In this example, the cash flows were selected in accordance with an IRR = .12. The recovery rates stabilized at ECRR = .4654 and TCRR = .5805. The significance of this differential becomes apparent when these rates are transformed, using a relationship similar to Eq (7), into estimates of the IRR. The TCRR converts to an $IRR_{TCRR} = .12$ and the ECRR results in an $IRR_{ECRR} = -.05$. Though this simple example seems to indicate the ECRR is a poor proxy for the TCRR, Stark reemphasizes his criticisms with a more rigorous proof to his hypothesis. (Stark, 1982:101-102)

After demonstrating the convergence of each rate, the difference between the two ratios was used to develop the following relationship (Stark, 1985:105):

$$TCRR_{n+j} - ECRR_{n+j} = a(g) \left((1+p_0)^{-1} - (1+p(g))^{-1} \right) - q(g) (1+p(g))^{-1} \quad (10)$$

where

- p = outlay on current assets
- j = years 0, . . . , n
- a(g) = weighted average of cash recoveries
- p(g) = weighted average of opening balances of current assets
- q(g) = weighted average of changes in current assets balance

Analysis of Eq (10) suggests these two rates will only be equal by coincidence unless the balance of current assets

for the typical firm project relative to an initial investment in current assets is equal for all years j (Stark, 1985:105). Stark used the following methodology to exemplify the ECRR's inept ability to serve as a proxy for the TCRR:

1. Randomly generate the input parameters (b , g , n , p , and r) used in Eq (7) and solve for the TCRR, ρ ;
2. Substitute the appropriate values for the variables in Eq (10) and solve for the ECRR;
3. Reevaluate Eq (7), this time, using the ECRR determined in Step 2, solve for r ; and
4. Compare the IRR estimated by Step 3, r , with the randomly generated r used in Step 1.

If the ECRR is an unbiased estimator of the TCRR, these two IRR values should be significantly similar within some predetermined degree of error. In two situations using different parameters to calculate TCRR, estimates of the IRR performed both poorly and well (Stark, 1985:106). Stark concludes that his analysis has demonstrated the TCRR is unobservable, there is no general direct conversion from the ECRR to the underlying TCRR, and use of the ECRR may introduce systematic biases into the estimated measure of firm performance.

Lee and Stark (1987) challenged the compatibility of Ijiri's definitions of cash recovery and investment with accepted capital budgeting principles. They caution that if Ijiri's designations are unsuitable there is the danger of

IRR estimates leading users to inappropriate conclusions and that the developing CRR literature is based on debatable foundations. Their evaluation begins with an algebraic analysis of Ijiri's cash flow system to demonstrate those parts which are supported by invalid assumptions. (Lee and Stark, 1987b:125)

From the mathematical modeling presented in the text, it becomes apparent that the Ijiri model not only has the capacity to introduce new classifications of cash flows but also to produce fundamentally different time patterns of cash flow recognition. (Lee and Stark, 1987:126)

These considerations may lead to a decision about project desirability based on Ijiri's approach which would differ from the decision based on a conventional capital budgeting model. The authors then use a fictional project, Project A, to provide a numerical example of Ijiri's model recommending rejection of a project when conventional cash-flow practices approve acceptance. Based upon the financial details created for Project A, the cash-flow data are used to calculate an $IRR_{cfd} = .21$ and the CRR is used to estimate an $IRR_{CRR} = .179$. This differential becomes significant if the firm's COC lies between these two rates. The reclassification referred to above:

will not of itself alter the reject/accept decision of a single project for a firm . . . however, a similar conclusion cannot be made on project rankings. (Lee and Stark, 1987:127)

Another fictional project, Project B, is introduced to demonstrate ranking differences between the two approaches.

Both projects are subjected to a $COC = .16$. Under the new conditions, Project A has an $IRR_{cfd} = .21$ and an $IRR_{CRR} = .174$ and Project B has an $IRR_{cfd} = .211$ and an $IRR_{CRR} = .171$. Conventional cash-flow analysis leads to the selection of Project B over Project A while Ijiri's model indicates the opposite conclusion.

Lee and Stark have shown Ijiri's model can produce investment evaluations differing from those derived from conventional DCF approaches and do not recommend its use in capital budgeting.

The implication of the analysis for the CRR is that it is fundamentally unsound . . . and the concepts . . . upon which the CRR is based are unsuitable for discounting. (Lee and Stark, 1987:130)

Ismail (1988) investigated the empirical approximation of the CRR which is used to solve Ijiri's and Salamon's theoretical models. He specifically questioned the use of a time series average to estimate the constant value which the CRR is believed to approach. For such a technique to be valid, the times series of a firm's annual CRRs in a twenty year period would have to be mean-reverting (see Appendix A) (Ismail, 1988:78). His testing of this condition failed to support the mean-reverting hypothesis. Ismail suggests:

the assumptions underlying the CRR are invalid, thus appearing to suggest that a necessary consequence of the assumptions underlying the CRR approach is that the CRRs should be stable over a twenty year period. (Ismail, 1988:87)

Rebuttal. Stark (1989) responded to Brief's (1985) and Ismail's (1988) assertions. In order to qualify his

response to Brief's statements, Stark outlines three definitions of economic performance and the assumptions necessary to use CRR-based estimates of the IRR as a measure of a firm's economic performance (Stark, 1989:278-281):

1. The rate of return being earned by the projects of the firm that are active in some specified period under consideration. There are two necessary assumptions: a) the firm is characterized as investing in a typical project in all the n years prior to, and in all but the final year of, the specified period; and b) investment flows over the n years prior to, and up to the penultimate year of, the specified period can be characterized as having grown at a constant rate;

2. The weighted average rate of return being earned by all of the firm's projects up to some specified date. There are three necessary assumptions: a); b) (same as in first definition); and c) the IRR of the projects completed prior to the specified period, taken together, is the same IRR as that earned by projects still active in the specified period.

3. The weighted average rate of return of all the firm's projects over its entire lifetime. There are three necessary assumptions: a); b); and c) (same as in second definition).

Brief stated the assumptions underlying the CRR approach imply a firm's "cash flows will grow at a constant rate into the indefinite future" (Brief, 1985:475). Given the three

sets of necessary assumptions above, corresponding to the three definitions of economic performance, it would appear, however, that Brief is incorrect in his assertions. (Stark, 1989:282)

Stark points out the assumptions which lead Brief to his conclusion are "sufficient but not necessary for the CRR approach to be valid under any of the definitions of economic performance (listed above)" (Stark, 1989:282).

The assumptions underlying the approach, for any of the definitions of economic performance, imply the CRR is constant over the relative period. Ismail studied the hypothesis that in a 20-year period after the life of a firm's typical project, $(n - (n+20))$, the presumed constant CRR should be mean-reverting. Stark comments that Brief's time orientation is of little relevance when judging the quality of CRR-based estimates of the IRR under the given definitions of corporate economic performance. Stark acknowledges the possibility of non-random measurement error in the CRR as suggested by Brief, but limits the measurement period to the commonly used length of five years (Gordon and Hamer, 1988; Griner and Stark, 1988; Ijiri, 1980; Salamon, 1982, 1985). (Stark, 1989:282)

In order to assess the mean-reverting quality of empirical CRRs, Stark studied the data for 1976 to 1980 for the 307 firms sampled by Griner and Stark (1988). The results of his analysis appear to question whether CRRs are likely to be mean-reverting over a five year period.

There seems to be a common element affecting the corporate CRRs. The existence of such a common

element, of itself, does not necessarily imply that CRRs are mean-reverting. This will depend on the behavior of the common element over time. (Stark, 1989:284)

Stark's conclusions suggest there is a level of measurement error in CRR-based IRR estimates which has not been previously considered. An understanding of the error's properties will clearly help decide under which circumstances the CRR best behaves as a proxy for the IRR.

It remains to point out, even if CRRs are not generated by a mean-reverting process over the five year period studied, nor the 20 year period studied by Ismail (1988), this does not, of itself, invalidate the CRR approach, as described (by Griner and Stark), to the estimation of corporate economic performance. (Stark, 1989:284)

Validation. One important problem associated with the conversion of CRRs into IRR estimates is the need to make assumptions about the cash-flow profile of the firm's typical (or composite) project. (Gordon and Hamer, 1988:514)

Gordon and Hamer (1988) extended previous analytical CRR work by incorporating concave cash-flow profiles, shown in Figure 4, into the CRR model. They then used their revised model to investigate whether IRR estimates derived using a concave profile are similar to those produced by the more simplistic profiles assumed in earlier studies.

"A small sample survey by the authors suggests that such (concave) profiles are more prevalent in practice than the profiles assumed in previous research;" 16 out of 23 firms chose Figure 4 as the representative cash-flow profile of their composite project (Gordon and Hamer, 1988:515). As Eq (11) indicates, the introduction of the concave cash-flow

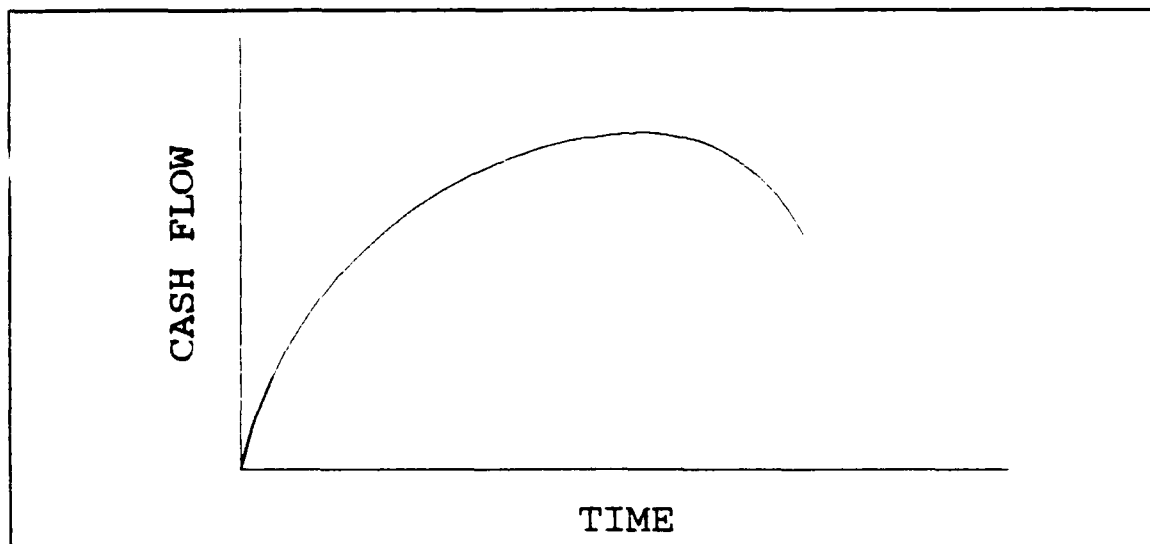


Figure 4. Concave Cash-flow Profile

profile preserves the relative simplicity of the multiplicative relationship between CRR and IRR (Gordon and Hamer, 1988:517):

$$\rho = \left[\frac{(1-pg)p^n g^n}{(1-p^n g^n)} \right] \left[\frac{i-B}{g-B} \right] \left[\frac{\frac{g^n - B^n}{g^{n-1}(g-B)} - \frac{nB^n}{g^n}}{\frac{i^n - B^n}{i^{n-1}(i-B)} - \frac{nB^n}{i^n}} \right] \quad (11)$$

where

$g, i, n,$ and p = (as defined for Eq (7))
 B = cash-flow parameter, $.50 < B < 1$

Cash inflows increase during the early years of the project's life, reach a maximum level, and then decrease. The year in which cash inflow is maximized depends directly on the value selected for B .

Gordon and Hamer estimated six conditional IRRs using Eq (11) and the average CRRs for the 20 firms examined

previously by Salamon and Ijiri (see Appendix F). The input parameter B was evaluated at $1/3n$, $1/2n$, and $2/3n$ under the economic lives of $n = 20$ and $n = \hat{n}$. B is calculated by solving Eq (12) (Gordon and Hamer, 1988:517).

$$k = -\frac{1}{\ln B} \quad (12)$$

where

$$k = 1/3n, 1/2n, 2/3n$$

Gordon and Hamer studied the behavior of Eq (11) when B was held constant for all firms and when it was allowed to vary from firm to firm. The most striking feature of Table 8 is that the IRR estimates derived for the revised model are highly associated with those derived from Salamon's (1982) model (Gordon and Hamer, 1988:518).

TABLE 8

PEARSON CORRELATION COEFFICIENTS (Gordon and Hamer, 1988:520)

	IRR(1)	IRR(2)	IRR(3)	IRR(4)	IRR(5)	IRR(6)
IRR(7)	.999					
IRR(8)	.995	.999				
IRR(9)	.992	.997	.930			
IRR(10)	.703	.682	.742	.998		
IRR(11)	.763	.750	.771	.987	.997	
IRR(12)	.773	.762	.770	.980	.994	.358

Note: IRR(1-6) are from Table 4. Input parameters for IRR(7-12) are as follows: IRR(7) $n = 20$, $k = 1/3n$; IRR(8) $n = 20$, $k = 1/2n$; IRR(9) $n = 20$, $k = 2/3n$; IRR(10) $n = \hat{n}$, $k = 1/3n$; IRR(11) $n = \hat{n}$, $k = 1/2n$; and IRR(12) $n = \hat{n}$, $k = 2/3n$.

"As noted earlier by Salamon (1982), there is a strong and positive correlation among the estimated IRRs based on the assumption $n = 20$; the lowest correlation between IRR(1-3) and IRR(7-9) is .93. IRR(4-6) and IRR(10-12), based on \hat{n} , exhibit a minimum correlation .858. A comparison of IRRs based on n against those based on \hat{n} reveals consistently lower correlations.

Thus, the results here are consistent with those of Salamon [1982, pp. 299-301] in that estimates of, or assumptions about, the useful life of firm projects are an important influence on the nature of the resulting IRR estimates. (Gordon and Hamer, 1988:518)

Gordon and Hamer demonstrated the CRR-based estimates of firm profitability appear to be fairly robust with respect to the choice of composite cash-flow parameters among increasing, level, decreasing, and concave profiles. They have provided further evidence that the distribution of IRR estimates within a sample of firms is insensitive to differing assumptions about the shape of the cash-flow profile as long as all firms share the same basic shape. (Gordon and Hamer, 1988:518)

Renovation. Salamon (1988), responding to comments by Stark (1987b) and Lee and Stark (1987), acknowledged the use of Eq (7) may lead to IRR estimates containing non-random measurement error. This bias occurs partly "because the CRR as calculated by (Eq (7)) is more a working capital recovery rate than a cash recovery rate" (Salamon, 1988:277).

Salamon recognizes there is much work to be done to refine the current state of CRR-based IRR estimates.

Griner and Stark (1988) reaffirm accusations made in earlier CRR literature stating Ijiri's cash-flow system and empirical definition of the CRR are inconsistent with conventional cash-flow practices. They proposed "a new and more general method for deriving estimates of economic performance from cash recovery rates" (Griner and Stark, 1988:295). Their first objective was to implement a method for estimating IRRs across a greater number of assumed cash-flow profiles.

Griner and Stark retain the steady state growth assumption common to all previous CRR work. The general form of their theoretical relationship between the CRR and the IRR is shown in Eq (13) (Griner and Stark, 1988:296):

$$CRR(t) = \frac{\int_0^N I(t-\tau) N(\tau, a_1, \dots, a_n) d\tau}{\int_0^N I(t-\tau) d\tau} \quad (13)$$

where

CRR(t) = cash recovery rate at time t
t = time of unit investment (t > 0)
N = total life of the project
I(t - τ) = investment expenditures through time
τ = life of investment to date, τ ∈ (0, N]
N(τ) = sequence of cash flow subsequent to a unit
of investment in the firm's productive asset

Assuming the functional form of N(τ) can be parameterised by n parameters, (a₁, ..., a_n), N(τ) can be represented by N(τ, a₁, ..., a_n).

If we specify values for $n - 1$ of these parameters (which we can assume, without loss of generality, to be a_2, \dots, a_n), Eq (13) can be written as a single equation in a single unknown, a_1 (Griner and Stark, 1988:296):

$$CRR(t) = \frac{\int_0^N I(t-\tau) N(\tau, a_1, \dots, a_n) d\tau}{\int_0^N I(t-\tau) d\tau} \quad (14)$$

Theoretical knowledge of $I(t - \tau)$, over the period $[t - N, t)$, combined with the empirically derived $CRR(t)$ estimate for the same period, implies Eq (14) can be solved for a_1 . Hence, a conditional estimate of the IRR, r , can be found by solving Eq (15): (Griner and Stark, 1988:296)

$$\int_0^N N(\tau, a_1, \dots, a_n) e^{-r\tau} d\tau = 1 \quad (15)$$

Since the relevant period of this approach is limited to $[t - N, t)$, it places no restrictions on investment behavior prior to time $t - N$ nor after t . This satisfies the assertions made by Brief (1985) in criticism of theoretical assumptions made by Salamon (1982) (Griner and Stark, 1988:296 fn 2).

The empirical portion of Griner and Stark's formulation begins with the redefinition of Eq (5), hereafter referred to as the Ijiri (IJ) rate. The new definition, hereafter referred to as the Lee and Stark (LS) rate, is

$$\text{Cash Recovery Rate} = \frac{\text{Entity Cash Flow}}{\text{Gross Fixed Assets}} \quad (16)$$

where

$$\begin{aligned} \text{Entity Cash Flow} = & (\text{total funds from operations}) \\ & + (\text{interest expense}) + (\text{sale of} \\ & \quad \text{investments}) + (\text{sale of} \\ & \quad \text{property, plant and equipment}) \\ & + (\text{change in accounts payable,} \\ & \quad \text{income taxes payable, and} \\ & \quad \text{other current liabilities} \\ & \quad \text{excluding short term debt}) \\ & - (\text{change in receivables,} \\ & \quad \text{inventories and other current} \\ & \quad \text{assets excluding cash}) \end{aligned}$$

and

$$\text{Gross Assets} = \text{average of opening and closing} \\ \text{gross fixed assets}$$

The CRR(t) is then approximated by a five-year average of firms' published financial data. Unlike the IJ rate, the LS rate calculates the ratio by dividing the five-year average of the numerator by the five year average of the denominator. "Such a procedure is keeping with the continuous time formulation used to estimate economic rates of return from cash recovery rates" (Griner and Stark, 1988:299).

Next, g and N are estimated. N is a five-year average of the \hat{n} parameter used by Salamon (1982, 1985, 1988). Since the IJ rate is defined using total assets (gross), Eq (3), and the LS rate is defined using gross fixed assets, Eq (16), estimation of g requires two separate rates. Both estimations were calculated using Eq (17) (Griner and Stark, 1988:300):

$$g = \frac{1}{5} \log \left[\frac{\text{Gross Assets}_{\text{end of period}}}{\text{Gross Assets}_{\text{beginning of period}}} \right] \quad (17)$$

Because of the steady state growth rate assumption, the flow of investment expenditures converts to Eq (18) (Griner and Stark, 1988:297):

$$I(t-\tau) = I_0 e^{g(t-\tau)} \quad (18)$$

This substitution is used to restate Eqs (14) and (15) as (Griner and Stark, 1988:297)

$$\text{CRR}(t) = gn(g) / (1 - e^{-gN}) \quad (19)$$

and

$$n(x) = 1 \quad (20)$$

where

$$n(x) = \int_0^N N(\tau) e^{x\tau} d\tau \quad (21)$$

The term $n(x)$ in Eq (20) is the Laplace transformation of $N(\tau)$.

It is useful to remember that, in essence, the Laplace transformation can be thought of as taking the present value of a function over its range (time in (this) case), using, in (Eq 21), x as the rate of interest. Thus, for example, $n(g)$ is the present value of the cash-flow profile $N(\tau)$,

defined on $\tau \in (0, N]$, using g , the rate of growth investment, as the interest rate. (Griner and Stark, 1988:297 fn 3)

Table 9 displays the eight functional forms of the cash-flow profiles used to estimate r . The parameter a , from Table 9,

TABLE 9
CASH-FLOW PROFILES (Griner and Stark, 1988:298)

<u>UPWARD SLOPING</u>	<u>PATTERN</u>
1. $N(\tau) = ae^{b\tau}$	exponential
2. $N(\tau) = a + b\tau$	linear
<u>SYMMETRICAL</u>	
3. $N(\tau) = a$	straight-line
4. $N(\tau) = a\tau$ on $(0, N/2]$ $= aN - a\tau$ on $(N/2, N]$	triangular
5. $N(\tau) = a \sin(\pi\tau/N)$	sine
<u>DOWNWARD SLOPING</u>	
6. $N(\tau) = a \cos(\pi\tau/2N)$	cosine
7. $N(\tau) = a - b\tau$	linear
8. $N(\tau) = ae^{-b\tau}$	exponential
(for 1., $b = .1$; for 2. and 7., $b = .005$; for 8., $b = .05$)	

is the single (unspecified) unknown referenced with respect to Eq (14).

Finally, the estimates of firms' growth rate, their average CRR, and the profile selected from Table 9 are used to solve Eq (19) for the unspecified parameter's value. Eq (20) uses the parameter value just determined and the

relationship stated in Eq (21) to calculate the estimate IRR, r . "This method can utilize any functional form for which a Laplace transformation exists for the cash-flow profile associated with an investment" (Griner and Stark, 1988:295).

Griner and Stark used the IJ rate and the LS rate along with all the patterns listed in Table 9 to generate eight different IRR estimates, IJ(1-8) and LS(1-8), for each of 307 firms. They used 1976-1980 financial data, drawn from Compustat tapes, in their sample for reasons of comparability with Salamon (1985, 1988).

Analysis of the estimated IRRs reconfirmed Salamon's (1982) conclusions that the ranking of firms by IRR is relatively insensitive to assumptions concerning the particular shape of the cash-inflow pattern as long as all firms are assumed to share a common profile.

Correlation between the different estimates of the economic rate of return, for a given definition of the cash recovery rate, range from 0.799 to 0.999 for the Lee and Stark rate and from 0.729 to 0.999 for the Ijiri rate (see Appendix G for a listing of all correlation coefficients). (Griner and Stark, 1988:301)

The correlations between the two definitions of the CRR are substantially lower; they range from 0.116 to 0.469.

Further examination of the results reveals the IRR estimates based on the LS rates are less correlated with the ARR than the IRR estimates based on the IJ rate. Since ARRs are considered to harbor significant levels of non-random error

as estimates of economic performance, the LS rate's low correlation is a desirable outcome.

This would suggest that, certainly, it is worth investigating further the use of the Lee and Stark rate as the internal rate of return estimates based on (the evidence that) this rate seem(s) to behave differently from both accounting rate of return and estimates derived from the Ijiri rate. (Griner and Stark, 1988:301)

At this point in their research, Griner and Stark switch their attention to the question of correlation between the estimates of the IRR and actual internal rates of return.

They begin by deriving an analytical relationship between the ratio of entity cash flow to investment expenditures (CF/IE) and the Laplace transformation of $N(\tau)$. This leads to an approximate linear relationship between CF/IE, the investment growth rate, and the IRR. To investigate whether estimates of IRRs display behaviors similar to actual rates of return, the linear relationship is tested by running a regression of Eq (22) (Griner and Stark, 1988:303-304):

$$CF/IE = \beta_0 + \beta_1 g + \beta_2 IRR^{(est)} + \mu \quad (22)$$

The null hypothesis used to test if the economic rates of return and their estimates have a positive and linear relationship declares β_2 will be significantly different than zero and positive. Test results confirm the null hypothesis for a one-tailed test to the 1 percent significance level (see Appendix H for regression results).

On the basis of the R^2 of the regressions, we might conclude the Lee and Stark cash recovery rate provides the basis for estimates of the internal rate of return with the highest correlation with the internal rate of return. (Griner and Stark, 1988:306-307)

Griner and Stark, after deriving a mathematical model capable of considering any functional form of the cash-flow profile for which a Laplace transformation exists, reconfirm the robust behavior of CRR-based IRR estimates with respect to the assumed cash-flow profile (Gordon and Hamer, 1988; Lee and Stark, 1987; Salamon, 1982, 1985). They redefined the empirical formulation of the CRR and showed it estimated IRRs which were substantially different than estimates based on Ijiri's definition of the CRR. The most significant aspect of their analysis was the testing for correlation between the various estimates of economic performance and the actual economic rates of return (internal rates of return).

Using a novel test, based on the analytically derived relationship between a cash-based ratio and the economic rate of return, we conclude that such is the case for all of (the) cash recovery rate-based methods of estimating the economic rate of return. Thus we would argue that one criterion that is necessary (but not sufficient) for any of the cash recovery rate-based estimators of economic performance to be suitable for use in cross-sectional studies of firm profitability is satisfied. (Griner and Stark, 1989:307)

To Be Done. "This area (systematic bias in cash recovery rate-based estimators of economic performance) of research has an obvious importance to the formulation of public policy" (Griner and Stark, 1989:308). Salamon (1985,

1988) has shown the factor of investment growth has an effect on the level of a firm's ARR. Griner and Stark (1989) suggest this effort is a potential lead in to the issue of systematic bias in estimators of economic performance. Salamon (1988) also recommends conducting empirical research on the time shape of project cash flows. Such work "could lead to more refined empirical estimates of firm IRRs than the conditional estimates relied upon in (previous studies)" (Salamon, 1988:287). Concentrated research on the question of systematic bias in the use of the CRR

can be seen as part of the process by which reliable research findings are generated as to the determinants of economic performance which then can constitute input into the public policy process determining competition policy. (Griner and Stark, 1989:308)

Précis. Ijiri considered the dichotomous relationship of investment analyses based on cash flow and project performance based on ROI a distinct hinderance to true measurements of firm profitability. As part of his efforts to emphasize the preeminence of cash flow-based accounting, he developed the CRR. It provide a measure of project or firm performance based on the same criterion used to select investments, the cash-flow data. CRR theory and models have grown quite extensively since Ijiri's introduction of the elementary corporate recovery factor relationship. Several economists have disputed the principles underlying the theory, yet its proponents persist. Consistent through all

of the models developed is the hope and belief that the CRR offers a unique opportunity to establishing a "compatibility between internal and external assessment (of a firm's) economic performance" (Stark, 1987a:11). Many challenges need to be answered. No longer among them, however, are questions doubting the practicality of the CRR as tool of economic and financial analysis.

Conclusions

As long as there are contrasting political and economic agendas to fulfill through government spending, the debate over the meaning and selection of the SDR will endure. Fortunately, the acute need for cost efficient and effective acquisitions compels a decision be made. The OMB has set the precedence that the SCC should reflect to some extent the OCC in the private sector. An understanding of capital budgeting by corporations is crucial to estimating the SCC.

In the 1990s, there has been a documented movement in support of the IRR as a budgeting technique. When confronted with its inefficiencies, corporate leaders have not abandoned the IRR, but rather modified its basic formulation to ensure its continued application. A nationally-derived IRR would provide a broad understanding of the potential earnings lost by industries as the government increases its investments. Despite mandates by federal agencies and prodding by public sector economists,

firm's published financial information stills does not provide sufficient data for such external assessments of profitability.

The rigorous development of CRR theories has made substantial contributions towards providing the external ability to measure corporate performance. Unfortunately, the theory is still constrained by the complexity and uncertainty in the environment it must be applied. Profound questions concerning the cash-flow parameter assumptions, the constant growth rate assumptions, and the introduction of systematic bias into numerical models must be resolved before unconditional, defensible estimations of IRRs can be derived.

III. Methodology

General Issue

Economists perpetually debate the SDR's selection criteria and advise updating the government's discounting policies; improvements would affect billions of dollars in defense acquisitions. Among those subscribing to the OCC school of thought, some advocate the ranking of projects according to their IRR. Building on these assumptions, the author proposes the formulation of a SDR based on a nationally-derived IRR. This version of the SDR could serve as a revision to the OMB-mandated DR or as an upper limit in the sensitivity analysis directed by AFR 173-15.

Determining corporation's and industry's IRRs is an perplexing task. Yuji Ijiri and Gerald L. Salamon have suggested the more readily measured CRR is a surrogate for the IRR. Current numerical models used to calculate CRRs must be validated under more complex and realistic scenarios. This chapter will describe the technique to be used to continue validation of the models developed by Salamon (1982) and Gordon and Hamer (1988).

Specific Issue

Current computational models demonstrate correlations between estimated IRRs and empirical CRRs. The purpose of this study is to validate these paradigms under fluctuating investment growth rates.

Terminology

A brief discussion is necessary to clarify the particular usage of the following general terms: project, firm, industry. A project is represented by an initial investment and subsequent cash recoveries. A firm is a collection of projects, each project is characterized by the same IRR. An industry is an collection of firms which share common cash-inflow and investment growth rate profiles.

Hypothesis and Investigative Questions

Research Hypothesis. Under the condition of a variable annual investment growth rate, cash recovery rate-derived estimates of internal rates of return, calculated by both models referred to above, will equal, to within an acceptable level of error, the corresponding directly calculated internal rates of return. The following criteria will be used to evaluate the hypothesis:

Investigative Questions.

1. Within each industry, do the numerical models estimate IRRs with enough precision to support the postulate of the CRR being a surrogate for the IRR?
2. Is there a correlation between the magnitude of the percentage error of each firm's estimated IRP and any of the parameters used to derive the estimate?
3. If a correlation exists, is it cogent enough to predict when the models' inaccuracies may exceed an acceptable percentage error level?

Justification

Several studies have validated the use of CRR-based methods of estimating firms' economic rate of return (Gordon and Hamer, 1988; Griner and Stark, 1988; Salamon, 1985). This thesis provides an extension to these studies by incorporating exponential and cyclical investment rates.

John Leslie Livingstone presented a 20-year study supporting substantial year-to-year fluctuations in firm expenditures on depreciable assets. Using least-squares estimates and regression statistics, he was able to model the investment patterns of a selection of corporations which were clients of Price Waterhouse and Company; all were members of the Fortune 500 list. His results, confirmed by R^2 values ranging between .556 and .898, produced four cyclical growth models, each including a sinusoidal function. (Livingstone, 1969:245+)

Gordon and Hamer surveyed 23 firms in order to determine their cash-inflow profiles. Of the 23 firms, 16 stated the concave profile (see Figure 4) depicted their composite project. They then derived an extension of Salamon's model (see Eq (7)) which included the more prevalent cash inflow profile (see Eq (11)). To validate their version of the model, Gordon and Hamer analyzed the same 20 firms Salamon used in his 1982 study. The lowest correlation observed between the two models is .93. (Gordon and Hamer, 1988:514+).

Considering the endorsement of CRR-based estimates of firm profitability and the statistical validation of the specific conditions included in this methodology, the approach proposed by this thesis is substantially justified.

Research Process

The primary tools of this research design are two empirical models that will compute discrete, deterministic IRR values based on varying inputs. The input parameters will be comprised of CRRs and the investment growth rates, inflationary rates, cash-flow parameters, and project economic lives used to generate randomized cash flows from which the CRRs are calculated. The output of the models will be used to test the hypothesis against actual IRRs from six industries; each industry will be comprised of 36 firms, each firm will be comprised of 30 annual investments in a typical project, and typical projects will have economic lives of 5, 10, and 15 years. The input parameters will be reflect authentic financial conditions based on historical data.

The input parameters for this study require firm-level financial information and will be gathered from secondary data sources. Cash-flow parameters, b and B , will be based on the results of studies by Gordon and Hamer, 1988, and Salamon, 1982. Annual investment growth rates, g_i ($i = 1, 2, \dots, 30$), will be derived from the annual real growth rates estimated by Salamon, 1982 (see Appendix D).

Once the data collection is complete, the primary steps for conducting this research are as outlined below.

1. For each of the industries listed below, develop a MathCAD template to generate project cash-flow parameters and firm annual investment growth rates (see Appendix I through Appendix L) (MathCAD, 1989):

a) Project b_s and B_s are randomly generated from a uniform distribution bounded by the limits established by their respective studies;

b) Project g_i s are randomly generated from a uniform distribution bounded by the limits of 20 firms' real growth rates and constrained to produce an 30-year compounded growth rate equivalent to a 30-year compounded growth rate based on the constant real growth rates listed in Appendix D; and

c) The four following industries, preceded by their identifiers to be used for the remainder of this document, were simulated:

1A - increasing, decreasing, or level b and a constant g_i ;

1B1 - increasing, decreasing, or level b and an exponentially increasing or decreasing g_i ;

1B2 - increasing, decreasing, or level b and a sinusoidal g_i ;

2A - concave cash inflow and a constant g_i ;

2B1 - concave cash inflow and an

exponentially increasing or decreasing g_i ; and

2B2 - concave cash inflow and sinusoidal g_i .

2. Develop a QUATTRO PRO spreadsheet that calculates each firm's annual CRRs, project IRRs, firm IRRs, and project and firm NPVs as a function of DR (see Appendix M) (QUATTRO, 1990). The project and firm NPVs are necessary to check the relevancy of their respective IRRs.

3. Develop a MathCAD template which calculates, for each firm, the mean of the annual CRRs to be used as an estimate of the value upon which the firm's CRRs would theoretically converge and also solves Eqs (7) or (11) for the CRR-based estimate of the firm's IRR (1x industries will use Salamon's model and 2x industries will use Gordon and Hamer's model) (see Appendix N through Appendix P).

4. Determine the percent error for each set of CRR-derived real IRR and its corresponding real IRR calculated directly from the cash-flow data.

5. Determine the frequency each of the models were able to accurately estimate the firm's IRR within the 5 percent and 10 percent error levels.

6. Examine trends in the level of percent error for correlation with some deterministically known parameter(s) of the generated cash flows.

7. Reject or do not reject the hypothesis based on the deductions drawn from the results of Steps 5 and 6.

Summary

The methodology described in this chapter will be used to depict the potential application of the numerical CRR-IRR models under more realistic economic conditions than those used to derive the relationships. With the use analytical software programs, the researcher will generate sets of randomized cash flows to simulate firm-level financial information. The characteristics of these cash flows become the two models' input parameters. The accuracy of the CRR-derived IRRs and the correlation of their percent error will determine the acceptance or non-acceptance of the research hypothesis.

IV. Findings and Analysis

Process

General. Based upon constraints obtained from financial reporting of existing firms and industries, a series of firm-level investment growth patterns and project-level cash-flow profiles were randomly generated from suitably apportioned uniform distributions. These random deviates were then used to simulate the firms of nine industries. Firms were created from 30 consecutive annual investments in a typical project. Typical projects had economic lives of 5, 10, and 15 years. The simulation of six industries was based on the following combinations of the two input parameters:

1A - increasing, decreasing, or level b and a constant g_i ;

1B1 - increasing, decreasing, or level b and an exponentially increasing or decreasing g_i ;

1B2 - increasing, decreasing, or level b and a sinusoidal g_i ;

2A - concave cash-flow profile and a constant g_i ;

2B1 - concave cash-flow profile and an exponentially increasing or decreasing g_i ; and

2B2 - concave cash-flow profile and a sinusoidal g_i .

The project and overall firm cash flows were evaluated to determine firm- and project-level IRRs, the stream of NPVs as a function of DR for both the firm and its typical

project, and the firm's annual CRRs. Because of the issue of relevant IRRs discussed in Chapter II, the NPV streams of each project and firm were examined to detect the existence of multiple IRRs and assist in the identification of the relevant value.

Finally, the firm's financial traits were used as inputs into Salamon's and Gordon and Hamer's IRR-estimation models. MathCAD's iterative computational algorithm solved Eqs (7) and (11) for r and ultimately produced the real IRR, r' . These estimated IRRs (IRR_e) were compared against the actual cash flow-based IRRs (IRR_a) by calculating the percent error between the experimental and theoretical values. Comparisons were conducted between the IRR_e s and both the project- and firm-level IRR_a s.

Several attempts were made to identify a relationship linking any variant of the input parameters to the magnitude of the percent error between the IRR_e and the firm-level IRR_a (PE_f) or the percent error between the IRR_e and the project-level IRR_a (PE_p). This analysis used Pearson Correlation Coefficients to quantify the degree of linear association between the variables studied.

Special Considerations. Many studies have set the precedence of using the mean of annual CRRs as an estimator of the value upon which a firm's CRRs will theoretically converge (Ijiri, 1979, 1980; Salamon, 1982, 1985, 1988; Stark, 1987a, 1987b, 1989). Since the data analyzed in this thesis begin with the creation of the simulated firms, it

was necessary to allow the firms' cash flows to reach a steady state before extracting the CRRs used in the convergence estimation. The stream of values used in the mean calculation never included annual CRRs occurring before the year of the final cash inflow of the firm's original investment.

Calculation of the IRR_e s under the investment growth conditions in the 1Bx and 2Bx industries required a small variation from the conventional solution of Eqs (7) and (11). After identifying the period of the firm's life to be examined, an annual IRR_e was computed for each year in that period as a function of its uniquely corresponding annual parameters. The final estimator of the IRR_a is the geometric mean of the annual IRR_e s.

Findings

Review of the percent error for each firm analyzed depicts per industry acceptance rates of the research hypothesis ranging from a low of 69.44 percent, industry 2B2, to a high of 95.37 percent, industry 2A, at the 5 percent error level (see Table 10). At the 10 percent error level, acceptance rates range from 85.19 percent, industry 1B2, to 98.15 percent, industry 2B1. Categorical acceptance rates, such as per industry, denote the percentage of the total number of firms within that category whose specified type of percent error, PE_p or PE_f , is less than or equal to the level of error under assessment. This thesis simulated

TABLE 10
ACCEPTANCE OF THE RESEARCH HYPOTHESIS
BASED ON PE_{fs} AND PE_{ps}

Industry	Level of Percent Error	Percentage of Firms			Total per Industry
		Project Life (years)			
		5	10	15	
1A	5	86.11	86.11	94.44	88.89
	10	86.11	91.67	97.22	91.67
1B1	5	91.67	88.89	63.89	81.48
	10	91.67	97.22	77.78	89.82
1B2	5	80.56	72.22	55.56	69.44
	10	86.11	88.89	77.78	84.19
2A	5	97.22	91.67	97.22	95.37
	10	97.22	94.44	97.22	96.30
2B1	5	100	97.22	86.11	90.74
	10	100	100	94.44	98.15
2B2	5	91.67	77.78	61.11	76.85
	10	100	94.44	80.56	91.67
Total per Project Life	5	91.20	86.11	76.39	
	10	92.98	94.44	87.96	

36 firms per industry for each of the three project economic lives. Table 10 indicates a definite bias towards firms in the 2x industries which invest in a 5-year project.

According to the fundamental assumptions incorporated into the CRR theory by Ijiri, the firm's economic rate of return will approach the project IRR (Ijiri, 1979:259+). Other researchers, while studying the relationships between a firm's accounting rate of return and its IRR,

"incorporated an assumption that firm IRR is equal to the IRR of the firm projects' regardless of the pace at which the projects have been acquired over time . . ." (Salamon, 1973:301). Several of the simulated firms' project IRR differs considerably from their firm IRR. When the research hypothesis is tested against the PE_p 's' magnitude independent of the PE_f 's', the acceptance rates are significantly different from those presented in Table 10 (see Table 11).

TABLE 11
ACCEPTANCE OF THE RESEARCH HYPOTHESIS
BASED ON PE_p 's

Industry	Level of Percent Error	Percentage of Firms			Total per Industry
		Project Life (years)			
		5	10	15	
1A	5	100	94.44	94.44	96.26
	10	100	100	97.22	99.07
1B1	5	100	88.89	63.89	84.26
	10	100	97.22	77.78	91.67
1B2	5	83.33	72.22	58.33	73.15
	10	91.67	88.89	77.78	87.04
2A	5	100	100	100	100
	10	100	100	100	100
2B1	5	100	97.22	88.89	95.37
	10	100	100	97.22	99.07
2B2	5	91.67	77.78	61.11	77.78
	10	100	94.44	80.56	91.67
Total per Project Life	5	95.83	89.35	80.09	
	10	99.07	96.76	88.43	

Table 11 depicts the same trend in high acceptance percentages as Table 10; however, the per industry and per project acceptance ranges are notably higher than those previously reported. A more significant change is the near perfect and perfect acceptance rates for the 1A and 2A industries, respectively. These two industries embody the firm's assumed financial conditions used in the derivation of the two IRR-estimation models. It is to be expected that their percent error levels would be minimized.

Unfortunately, when the models are applied under realistic conditions, the project-level IRR_a s are not known. Acceptance rates based solely on firm-level IRR_a s differ considerably from those listed in Table 11 (see Table 12). As in the two previous tables, the larger acceptance rates are biased towards 2x industries with 5-year typical projects. The similarities between Tables 10 and 12 indicate the major role the level of PE_f plays in determining the reported acceptance rates.

A direct contrast of the acceptance rates for both models, on a per economic life basis, clearly displays the superior estimation capabilities of Gordon and Hamer's model over Salamon's model under the simulated conditions of this thesis (see Table 13). Gordon and Hamer's model outperformed Salamon's model for each project life, percent error level, and error type tested.

If CRR theory and, specifically, the IRR-estimation models are to truly provide reliable measures of firm

TABLE 12
ACCEPTANCE OF THE RESEARCH HYPOTHESIS
BASED ON PE_{fs}

Industry	Level of Percent Error	Percentage of Firms			
		Project Life (years)			Total
		5	10	15	
1A	5	86.11	86.11	94.44	88.89
	10	86.11	91.67	97.22	91.67
1B1	5	91.67	94.44	63.89	83.33
	10	91.67	97.22	86.11	91.67
1B2	5	80.56	72.22	55.56	69.44
	10	86.11	88.89	77.78	84.26
2A	5	97.22	91.67	97.22	95.37
	10	97.22	94.44	97.22	96.30
2B1	5	100	100	86.11	90.74
	10	100	100	94.44	98.15
2B2	5	91.67	77.78	61.11	76.85
	10	100	94.44	80.56	91.67
Total per Project Life	5	91.20	87.04	76.39	
	10	93.52	94.44	87.96	

performance, the error and limitations inherent in their predictions must be realized and understood. One means of identifying possible constraints is to uncover a relationship between the PE_f and the input parameters of Eqs (7) and (11).

After studying the correlation between numerous combinations of the input parameters and the level of percent error, no consistently significant relationships

TABLE 13

AN ACCEPTANCE RATE COMPARISON
OF SALAMON'S AND GORDON AND HAMER'S MODELS
BASED ON PE_f s AND PE_p s

Level of Percent Error	Model	Percentage of Firms by Project Life					
		5		10		15	
		PE_f	PE_p	PE_f	PE_p	PE_f	PE_p
5	1 ^a	86.11	88.89	82.41	93.52	71.30	85.11
	2 ^b	96.30	99.07	88.89	96.30	82.41	90.74
10	1	94.44	98.15	87.04	95.37	75.00	84.26
	2	97.22	100	91.67	98.15	84.26	92.59

Note: ^aSalamon's model; ^bGordon and Hamer's model.

were observed. The parameters and combinations which occasionally exhibited significant levels of correlation are listed in Table 14. Pearson Correlation Coefficients greater than .9, such as g/r 's in industry 1A for projects with $n = 5$ years, indicate a notable association between the input parameters and the PE_f . By exploring these types of trends, it may be possible to develop acceptance criteria to evaluate IRR_e s.

Analysis

Model Performance. The author is unable to diagnose the causes for Gordon and Hamer's model outperforming Salamon's model but is able to discern certain characteristics varying between the 1x and 2x industries. Evaluation the theoretical equality between the firm- and project-level IRRs reveals a 49.21 percent compliance with

TABLE 14
PE_f PEARSON CORRELATION COEFFICIENTS
FOR ALL FIRMS

INDUSTRY							
n	r ^a	CRR	g	s ^b	g/r	s/c ^c	g/r/s ^d
<u>1A</u>							
5	-.5635	-.4845	.3129	.1404	.9763	.4700	-.0258
10	-.5657	-.4152	.4443	.1272	.8567	.4711	.4384
15	-.1735	.0432	.2135	.3441	.3006	.3752	-.1406
<u>1B1</u>							
5	-.5449	-.4891	.1142	.0142	.8768	.2073	-.0432
10	-.2998	-.4037	.2183	.5659	.2763	.7495	-.1867
15	-.5288	-.5437	.3681	.4047	.4375	.9420	-.1629
<u>1B2</u>							
5	-.4571	-.3990	.3630	.3488	.7679	.5641	-.0524
10	-.1572	-.3065	.4007	.7028	.2158	.9044	-.0874
15	-.4867	-.4406	.3801	.4324	.6534	.9195	-.2040
<u>2A</u>							
5	-.5150	-.3491	-.0707	-.1133	.9291	.1522	.0917
10	-.5248	-.2856	.1891	-.0735	.8958	.2657	-.0506
15	-.4835	-.0875	-.2122	.1053	.7671	.0353	-.0363
<u>2B1</u>							
5	-.3286	-.2557	.1895	.2543	.5913	.3329	-.1596
10	-.5668	-.3611	.6023	.2321	.5643	.7514	-.2331
15	-.4072	-.2423	.3374	-.0317	.7298	.5676	.0660
<u>2B2</u>							
5	.3556	.4750	.6179	.6812	-.2287	.7263	-.1546
10	-.2724	-.2245	.4514	.4204	.2641	.8650	-.2150
15	-.1766	-.2001	.4092	.1997	.1295	.9101	-.0509

Note: ^afirm-level IRR_e; ^bstandard deviation of firm's annual CRRs; ^cstandard deviation / CRR; and ^dannual growth rate / firm IRR_e / standard deviation of annual CRRs.

the assumptions within the 1x industries and a 75 percent compliance within the 2x industries. The cash flows of the

1x industries exhibited various qualities that would contribute to the low percentage of conformance.

Salamon proposed the theoretical equality of the two IRRs to be incorrect. Specifically, he has shown

that if the growth rate in gross investment is greater than or equal to the IRR of firm projects then the IRR of the firm is not equal to the IRR of firm projects; and in fact, the IRR of the firm is not even defined. (Salamon, 1973:301)

Twelve and one-half percent of the firms in the 1x industries with PE_{fs} greater than 10 percent have an investment growth rate larger than the corresponding project IRR; no 2x-industry firms displayed this condition.

Occasionally, a firm-level cash flow within the 1x industries experienced multiple transitions between positive and negative annual values. The firms in which this peculiarity occurred were characterized by the combination of a steeply declining cash-flow profiles and a large annual growth rate. Apparently, the concave cash-flow profile associated with Gordon and Hamer's model did not allow cumulative annual recoveries to be out paced by the annual growth in gross investment. None of the 2x industries' firms encountered the oscillation described above.

Table 14 reveals a higher correlation between PE_{fs} and both IRR_{es} and CRRs in the 1x industries than those for the corresponding 2x industries. The majority of the firms with PE_{fs} greater than 5 percent have IRR_{es} less than .2000. Average IRR_{es} for the 1x industries are approximately 12 percent lower than those of the 2x industries.

Subsequently, more 1x industries have IRR_e s below .2000. This contrast between IRR_e magnitudes seems to be a function of the 2x industries' concave cash flows providing significantly larger recoveries in the life of a project which result in larger CRRs. The larger CRRs of the 2x industries tend to produce relatively larger IRR_e s.

Contrary to the relative performance of the two models, the pattern of the firms' annual CRRs for the 1x industries were consistently more convergent than those of the 2x industries. The degree of convergence is measured by the magnitude of the standard deviation of a firm's annual CRRs around the mean value used in the solution of Eqs (7) and (11). On a corresponding industry basis, the 1x standard deviations ranged from 5 to 17 percent lower than the 2x standard deviations. Tighter convergence of 1x CRRs would lead to the conjecture that the mean CRR of each firm would more closely estimate the theoretical convergent value of the CRR and improve the accuracy of the IRR_e . However, the results of Table 13 do not support such conclusions.

This discussion is not offered to insinuate the superiority of Gordon and Hamer's model over Salamon's model. It is only an exploration of reasons why, under the restrictive conditions of this simulation, the IRR_e s calculated by Eq (11) more reliably estimate their respective IRR_a s.

Acceptance Criteria. In an ideal situation, an investigator applying an IRR-estimation model would be able

to predict the reliability of estimates based upon a set of established rules. Salamon's relationship between a firm's IRR and its investment growth rate is an example of the type of rules needed. Prompted by correlations depicted in Table 14, the author derived the following algorithm for predicting the accuracy of IRR_e s calculated by Eqs (7) and (11) (see Figure 5):

1. If the annual growth rate is greater than the IRR_e , then do not accept the IRR_e ;

2. If the ratio of annual growth rate to IRR_e is greater than 6.5, then do not accept the IRR_e ;

3. If the ratio of the standard deviation of the firm's annual CRRs to the mean CRR is less than .02, then accept the IRR_e ; and

4. If the ratio of the standard deviation of the firm's annual CRRs to the mean CRR is greater than .02, then test the estimate's parameters against the rules listed in Table 15.

Based on the 648 IRR_e s computed using Eqs (7) and (11), this acceptance algorithm erroneously accepted 10, 1.54 percent, IRR_e s with PE_f s greater than 5 percent, and mistakenly did not accept 27, 4.17 percent, IRR_e s with PE_f s less than 5 percent. Table 16 lists acceptance rates after the IRR_e s were evaluated by the acceptance algorithm.

Almost every non-100 percent acceptance rate from Table 12 improved after the algorithm was applied. To compare percentage of firms accepted between Tables 12 and 16, it is

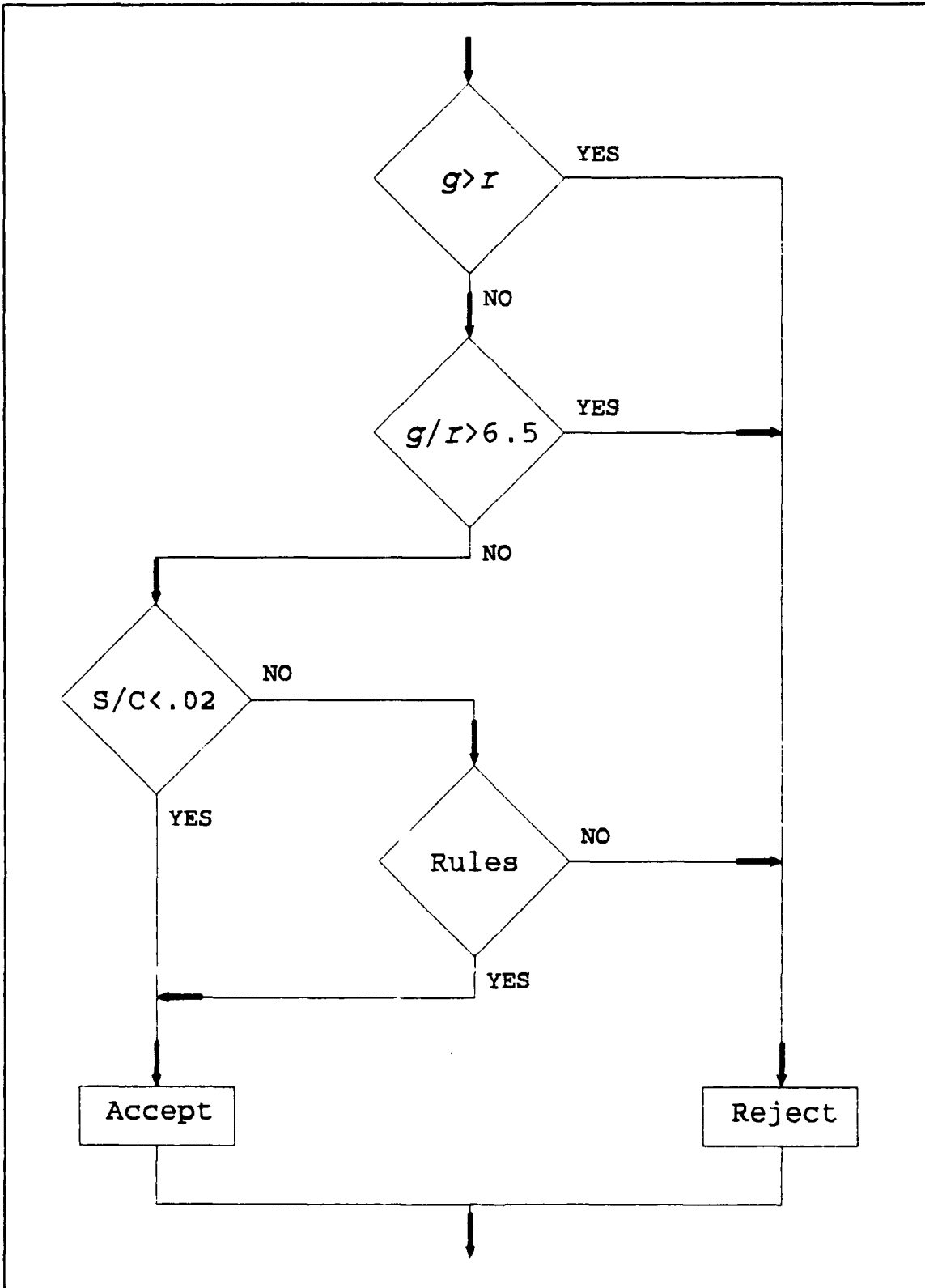


Figure 5. Acceptance Algorithm

TABLE 15
ACCEPTANCE RULES FOR IRR_e WITH $S/C > .02$

	<u>If</u>	<u>and</u>	<u>then</u>	<u>else</u>
1.	$S \leq .003$		accept	
2.	$.003 < S \leq .004$	$g/r/S < 644$	accept	reject
3.	$.004 < S \leq .005$	$g/r/S < 560$	accept	reject
4.	$.005 < S \leq .006$	$g/r/S < 484$	accept	reject
5.	$.006 < S \leq .007$	$g/r/S < 434$	accept	reject
6.	$.007 < S \leq .008$	$g/r/S < 324$	accept	reject
7.	$.008 < S \leq .011$	$g/r/S < 232$	accept	reject
8.	$.011 < S \leq .020$	$g/r/S < 165$	accept	reject
9.	$.020 < S \leq .029$	$g/r/S < 80$	accept	reject
10.	$.029 < S \leq .041$	$g/r/S < 45$	accept	reject
11.	$.041 < S \leq .061$	$g/r/S < 30$	accept	reject
12.	$.061 < S \leq .071$	$g/r/S < 20$	accept	reject
13.	$.071 < S \leq .100$	$g/r/S < 15$	accept	reject
14.	$.100 < S \leq .200$	$g/r/S < 8$	accept	reject
15.	$.200 < S$	$g/r/S < 4$	accept	reject

important to recognize the effect of not accepting some of the IRR_e s. Since the total firms evaluated per industry and project life are less than the 36 previously considered, one IRR_e with a PE_f greater than 5 percent will reduce the acceptance rate more in Table 16 than in Table 12.

In each of the industries, and for all project lives, the algorithm did not accept the majority, if not all, of the IRR_e s with PE_f s greater than 5 percent. The most that were accepted is three; once in the 1B2 industry and once in the 2B2 industry. Both are for 15-year projects. The bias towards the shorter lived projects in the 2x industries still exists but can be considered negligible in comparison to Tables 10, 11, 12, and 16.

TABLE 16

ACCEPTANCE OF THE RESEARCH HYPOTHESIS
BASED ON PE_f s AND THE ACCEPTANCE ALGORITHM

Industry	Level of Percent Error	Percentage of Firms			
		Project Life (years)			Total
		5	10	15	
1A	5	100	96.88	94.29	96.97
	10	100	100	97.14	98.99
1B1	5	100	100	100	100
	10	100	100	100	100
1B2	5	100	100	85.00	95.77
	10	100	100	90.00	97.18
2A	5	100	97.06	100	99.04
	10	100	100	100	100
2B1	5	100	100	100	100
	10	100	100	100	100
2B2	5	100	96.30	91.3	96.25
	10	100	100	100	100
Total per Project Life	5	100	98.37	95.63	
	10	100	100	98.13	

Of the 126 correlations reported in Table 14, 77, 61.11 percent, were unchanged or decreased in magnitude after application of the acceptance algorithm (see Table 17). The consistently high correlation between the ratio g/r and PE_f in Table 14 is not repeated in Table 17. None of the parameters in Table 17 display significant trends in relationship to the level of percent error.

TABLE 17
PE_f PEARSON CORRELATION COEFFICIENTS
FOR MODIFIED INDUSTRIES

INDUSTRY							
n	r ^a	CRR	g	s ^b	g/r	s/c ^c	g/r/s ^d
<u>1A</u>							
5	-.6212	-.5411	.4014	-.2891	.8070	-.1137	.7754
10	-.2854	-.3170	.1106	.2953	.4450	.4391	-.1041
15	-.0316	.1549	.2664	.2946	.1342	.2794	-.1326
<u>1B1</u>							
5	-.6161	-.5558	.1614	-.1364	.7679	-.0753	.1131
10	-.4772	-.5083	.0677	.2680	.5148	.6020	-.1361
15	-.4116	-.5302	-.1211	.0988	.4314	.4537	-.0989
<u>1B2</u>							
5	-.1897	-.2416	.3864	.7605	.1740	.7692	-.1820
10	.1574	.1862	.2887	.7461	-.1327	.7740	-.0503
15	.2318	-.1582	.5159	.7941	-.1918	.8900	-.2566
<u>2A</u>							
5	-.1817	-.1556	.0570	.2247	.2657	.3420	-.1807
10	-.2831	-.2109	-.2634	.3599	.2993	.5206	-.0954
15	-.3877	-.1644	.0186	-.0674	.9432	.1350	-.0227
<u>2B1</u>							
5	-.3286	-.2557	.1895	.2543	.5913	.3329	-.1596
10	-.6312	-.3794	.6024	.2286	.7751	.7652	-.2353
15	.1009	.0043	.5094	.2621	-.0296	.3180	-.1536
<u>2B2</u>							
5	.4789	.5716	.4968	.5341	-.2676	.5244	-.1329
10	-.1590	-.0113	.1241	.4194	.1687	.7276	-.1566
15	-.2806	-.1925	.1280	.2492	.4099	.5386	.3801

Note: ^afirm-level IRR_e; ^bstandard deviation of firm's annual CRRs; ^cstandard deviation / CRR; and ^dannual growth rate / firm IRR_e / standard deviation of annual CRRs.

A test for normality of the 126 Pearson Correlation Coefficients in Table 17 results in a Shapiro-Francia statistic, an approximation of the Wilk-Shapiro normality

statistic, equal to .9793; values above .9000 are considered sufficient evidence to support an assumption of normality (Statistix, 1990). Based on this test statistic, the sample of correlation coefficients is indicative of a normally distributed population characterized by the descriptive statistics $\mu = .1279$ and $\sigma = .03415$.

Error. Two types of error must be considered: analytical and empirical. Particular analytical factors in the simulation may have contributed to error in the estimations. Salamon (1988) acknowledged the use of Eq (7) may lead to IRR estimates containing non-random error partly because of the required use of an average CRR in Eq (7). The accuracy of the mean CRR estimate of the theoretical convergent value is closely linked to decisions made about the steady state of the firm's cash flows and the particular time period analyzed.

Hsiao and Smith pointed out the sensitivity of IRR_{es} is a function of project life, the percentage of the original investment recovered in the first year, and the cash-flow parameter. They suggested projects with long lives, large initial recoveries relative to the investment, and quickly declining cash-flow profiles are among those least susceptible to the error inherent in estimation models. Needless to say, all of these optimal conditions were breached during the randomized generation of the input parameters.

Several procedures in the simulation provided the potential for sizable measurement error (see Appendix A). The calculation the project-level IRR, firm-level IRR, and the IRR_e could not be done directly and depended on iterative solutions. Sources of variability include dependence on the initial value guessed for the unknown and the characteristic tolerances of the mathematical functions used in the computations. Since both of these parameters are selected by the investigator, there is a significant opportunity to introduce error into the process.

Occasionally, the iterative calculation of Eqs (7) or (11) would not converge. To secure a solution, the period from which the annual parameters were extracted had to be manipulated. As a result, the IRR_e for a certain time frame included annual parameters from outside the period being analyzed.

The final calculation of an IRR_e included the geometric mean of two values: average CRR, average of the annual IRRs for the xBx industries. This combination compounded the effect of errors due to averaging.

Summary

Despite the computational intricacies and idiosyncracies involved in the simulation, the experimental results provide some new insights into the versatility of existing numerical, IRR-CRR, estimation models. Though derived under the assumption of constant investment growth

rates, Salamon's and Gordon and Hamer's models are still capable of producing reliable IRR_e s when applied under more complex and realistic investment patterns. The behavior of these models may or may not be judged acceptable as a function of the accuracy required by the investigator.

V. Conclusions and Recommendations

Conclusions

Discounting, SDR, and IRR. Selection of the DR is perhaps the single most critical decision to be made when beginning a cost-benefit analysis of alternative proposals. In the realm of public investments, discounting practices directly impact the SCC. Chapter II has made it incontrovertibly evident that economists remain divided with regards to the intention of the SDR and its appropriate value. As long as the governing federal agencies support a SDR based primarily on an OCC approach, the prospect exists for the IRR, and subsequently the CRR, to contribute to the determination of the SCC. Private industries are steadily shifting towards capital budgeting techniques formulated around the IRR.

CRR. The idea of the CRR, introduced by Ijiri, has provided theoretical principles

necessary in the assessment of economic performance by means of cash-flow data, particularly with a view to ensuring a compatibility between internal and external assessment. (Stark, 1987a:i)

Many economists, including Brief, Gordon, Griner, Hamer, Ismail, Lee, Salamon, and Stark, have contributed to the extension of Ijiri's reemphasis of cash-flow accounting and the introduction of his corporate recovery factor.

Development of CRR theories has matured to the stage of

evaluating the criterion necessary to accept the CRR-based estimators of IRRs.

Griner and Stark provided evidence of essential correlations between CRR-based conditional IRRs and true economic rates of return. Thus, they

argue that one criterion that is necessary (but not sufficient) for any of the cash recovery rate-based estimators of economic performance to be suitable for use in cross-sectional studies of firm profitability is satisfied. (Griner and Stark, 1988:307)

Stark has also addressed the challenges, raised by Brief (1985) and Ismail (1988), to the necessary assumptions underlying the CRR approach to economic performance estimation. He acknowledges the potential measurement error in CRR estimates of IRRs and urges researchers to develop an understanding of the impact of such error. This type of research is important in the formulation of public investment policies.

To the extent that competition policy is based upon research findings on the determinants of economic performance which rely, at least in part, upon estimates of economic performance derived from accounting data, assessing the suitability of such estimates has clear value. This line of research, thus, can be seen as part of the process by which reliable research findings are generated as to determinants of economic performance which then can constitute input in the public policy process . . . (Griner and Stark, 1988:308)

"Despite the existence of imperfections in the cash recovery rate approach, it is a useful and practical tool of economic and financial analysis" (Stark, 1987a:17).

Simulation. Both Salamon's and Gordon and Hamer's IRR-estimation models performed rather robustly when applied under economic conditions other than those used in their derivation. However, trends illustrated under the Total heading in Table 12 depict apparent sensitivity to the degree the constant rate assumptions are transgressed. The xB2 industries, characterized by the most drastic deviation from the constant investment growth rate condition, contain the largest percentage of inaccurate IRR_es. Each investigator must determine the acceptability of these acceptance rates.

The unanticipated precision of the acceptance algorithm is a welcome surprise. Specific ranges used in the conditional statements are clearly rough estimates. Several limits were determined according to the behavior of only one or two firms. Nonetheless, the values in Table 16 affirm the algorithms effectiveness and dependability.

Analysis of the results listed in Table 17 imply a random nature for the error in the IRR-estimation models. This is not meant as a contradiction to Griner and Stark's or Salamon's discussions of non-systematic error in IRR_e calculations nor as an insinuation of non-random error in the simulation. The prediction error suggested in CRR literature refers to biases which are a function of estimating recovery rates from firms' published financial statements. Because this thesis uses simulated cash-flow data, this type error has not been introduced into the

calculations. The pattern of correlations shown in Table 14 indicate the existence of bias in the estimation processes. Fortunately, application of the acceptance algorithm reduces any non-systematic error to a level indicative a normally distributed random variable.

An interesting byproduct of the simulation provides some insight into the actual investment growth rate of a firm. As can be seen in the CRR vs YEAR plots in the IRR1A(1B, 2A, 2B).MCD MathCAD templates, the annual CRRs follow a pattern similar to the investment growth rate pattern of the firm under investigation (see Appendix N, O, and P). Realization of the investment pattern coupled with an understanding of acceptance-rate trends, such as those in Table 16, provides a baseline for gauging the accuracy of particular model applications. Considering the two models' performance documented throughout Chapter IV, the author recommends accepting the research hypothesis.

Recommendations

Theoretical. This subsection is presented as a synopsis of the areas for future study suggested by the professionals and academics involved in CRR research. There exists "relatively little published empirical evidence as to the ability of the cash recovery rate approach to measure economic performance" (Stark, 1987a:16). The efforts of this thesis are intended to address this dearth.

Everyone involved in CRR research realizes the conditional constraints levied by the lack of information surrounding the direct estimation of project cash-flow profiles. A strong need exists for studies which would "provide insight on the reasonableness of the common cash-flow parameter assumption made in (Salamon's 1988 work) and much prior work" (Salamon, 1988:288). Because of econometric difficulties, Salamon suggests research may have to proceed in an indirect manner. Fisher and McGowan propose, "in principle, this parameter could be estimated from a regression of cash recoveries on a distributed lag of past investments" (Salamon, 1988:275).

Before estimators of economic performance are confirmed as valid measures of firm profitability, the issue of systematic bias must be more thoroughly addressed. Salamon has begun research in this area by examining the properties of measurement error in ARRs. Griner and Stark (1988) recommend extending this type of research to include the effect of nondepreciation accruals. As these studies provide more understanding, their methodologies should then be applied to CRR-based IRR_e s. Forthcoming literature by Griner and Stark and Stark and Gordon deals with these points.

Experimental. Certain modifications to this thesis' methodology would improve the reliability of numerical simulations used to study CRR-IRR relationships:

1. When the iterative solution of estimating models

similar to Eqs (7) and (11) requires changes in parameters, ensure all inputs to the model are changed according, such as all parameters originating from the same time period;

2. Conduct a sensitivity analysis to determine the computational error inherent in the iterative algorithms as a function of tolerance levels and initial guesses;

3. Extend the life of the firm as far as necessary to allow the annual CRRs establish a converging pattern. This will improve the accuracy of the estimation of the theoretical convergent value; and

4. Integrate statistical mean calculations, whenever possible, instead of geometrical averages.

A logical extension of this thesis would be to study the impact of violating the constant inflation rate assumptions on the performance of both models. The following recommendations represent opportunities to respond to the scarcity of published empirical data denoted by Stark:

1. Conduct a simulation similar to that used in this study to compare acceptance rates and to continue evaluating and formulating the acceptance algorithm;

2. Since the majority of IRR_e s not accepted by the algorithm established in Chapter IV were less than .2000, constrain a simulation to focus primarily on cash flows that generate relatively lower CRRs;

3. Specifically study the correlational properties of IRR_e s with unacceptable percent error levels;

4. Examine financial parameters that lead to a significant differences between firm- and project-level α s in hopes of developing a predictive capability;

5. Investigate the link between a firm's investment growth rate pattern and the pattern of annual CRRs identified by the author; and

6. While working with a database of simulated cash flows, explore the behavior of conditional IRR_e s in an attempt to assess the actual impact of the presently necessary assumptions about the cash-flow profile of a firm's typical project and about industries as a whole.

Based on the encouraging results of this thesis, work should continue to develop the innovative strategy of using a national-level, CRR-derived IRR as an input in updating SCC and cost/benefit analyses. A viable next step in this strategy would be to calculate a national-level IRR_e and compare it to accepted measures of the SDR based on the OCC approach. This effort could be carried out in series or parallel with recommendation six from above.

Appendix A: Glossary of Technical Terms

Accounting rate of return - Net income before taxes divided by total investment. Often considered the true underlying economic rate of return.

Cash inflows - Any current or expected revenues or savings directly associated with an investment.

Cash outflows - The initial cost and other expected outlays associated with an investment.

Cash recovery rate - The percentage calculated by comparing all related cash inflows from an operation (investment) against the gross assets related to the operation (investment).

Constructive cash flow - Cash flows which are not accompanied by cash receipts or disbursements.

Cost of capital - The weighted average of the cost of debt capital and equity capital; equals the rate of return that a firm must earn in order to satisfy the demands of its owners and creditors.

Discount rate - The minimum desired rate of return on an investment. Synonymous with the required rate of return.

Discounted cash flow - Any monies (past/present/future) that has been transformed by a predetermined rate to equal the monies of a single preselected period of time.

Discounted cash flow rate - The rate at which the present value of all cash inflows and outflows becomes zero. Synonymous with internal rate of return.

Discounting - A budget technique that takes into account the time value of money by converting monies to be invested or received in the future into a present value.

Economic life - The time during which benefits from a project may reasonably be expected to accrue.

Economic performance - Economic rate of return (internal rate of return) being earned by the organization under consideration.

Financing activities - Cash is received with a promise to repay in the future. Transactions include financing (cash inflow) and repayment (cash outflow). Repayment is comprised of the original financing (refunding) and cash outflows above refunding (premium).

Fungibility - The ability to reallocate the returns on an investment, over time, and at some determinable rate of return.

Gross assets - Average value of the historical, undepreciated cost of all assets of a related objective.

Internal rate of return - The rate of interest at which the present value of expected cash inflows from an investment equals the present value of expected cash outflows of the project. Synonymous with the discounted cash flow rate.

Intertemporal marginal rate of transformation - The most efficient rate at which society is able to transform resources today into resources tomorrow.

Intertemporal marginal rate of substitution - The rate which society is willing to forgo resources today for resources tomorrow, leaving utility unchanged.

Investment activities - Cash spent in anticipation of its recovery in the future. Transactions include investment (cash outflow) and recovery (cash inflow). Recovery is comprised of the original investment (recapture) and cash inflows above recapture (return).

Mean-reverting - A parameter from a sample of data is said to be mean-reverting if it is stable other than through reasons of random error.

Measurement error - The element in an estimator that is not explained by variations in the population statistic.

Net present value - The difference between the present values of an investment's expected cash inflows and outflows.

Opportunity cost of capital - The value which funds channeled into government spending would have earned if left in the private commercial sector.

Percent error - A measure of the variation between corresponding theoretical and experimental values. Calculated as the absolute value of the difference multiplied by the ratio of 100 over the theoretical value.

Present value - The value today of an amount of money to be received or paid in the future.

Proper cash flows - Cash flows which coincide with cash receipts or disbursements.

Residual cash flows - Cash flows which are recognized only after cash receipts and disbursements are netted.

Second-best world - A functioning economic system that is characterized by distortions in the form of externalities, market power, market failure, and government sacrifice of efficiency for equity.

Shadow price of capital - The present value of the stream of consumption benefits associated with one dollar of private investment discounted at the social rate of time preference.

Social discount rate - The rate used to discount the future cash flows associated with government investments (synonymous with social cost of capital).

Appendix B: Corporate Recovery Rates (Ijiri, 1980:56)

	Recovery Rates										Avg
	1972	1973	1974	1975	1976	1977	1978				%
Alcoa	8.2	7.8	9.2	6.4	8.4	8.4	10.4				8.4
American Brands	10.2	10.8	11.4	10.5	9.3	10.5	11.1				10.5
American Can	6.5	8.0	8.5	8.1	8.4	8.6	8.1				8.0
Bethlehem Steel	6.3	7.2	9.0	8.1	7.2	2.4	8.4				6.9
Chrysler	9.1	9.6	4.9	2.8	11.3	6.3	9.7				7.7
DuPont	10.8	12.7	10.2	9.1	11.3	11.9	12.7				11.2
Esmark	8.6	9.5	11.6	11.2	13.9	12.1	11.2				11.2
General Electric	9.9	10.1	10.4	10.8	10.7	10.5	11.2				10.5
General Foods	12.1	10.8	10.7	11.3	11.2	11.7	9.8				11.1
General Motors	14.6	15.2	9.9	12.0	16.1	15.8	16.7				14.3
Goodyear	8.4	8.1	7.8	7.1	7.3	8.7	8.7				8.0
Gulf Oil	10.7	13.3	12.9	10.5	11.5	8.9	9.8				11.1
IBM	18.2	19.0	18.8	18.3	18.7	19.5	19.9				18.9
International Harvester	6.8	7.0	8.1	8.6	9.4	10.4	6.9				8.2
International Paper	8.4	10.5	12.3	13.3	12.0	11.4	11.4				11.3
RCA	14.0	13.9	12.0	10.3	13.7	15.1	15.6				13.5
Texaco	9.7	12.6	14.2	6.0	8.0	9.9	9.6				10.0
United Technology	7.1	6.7	8.9	8.6	13.0	9.3	9.1				9.0
U.S. Steel	4.3	6.3	9.5	7.4	9.4	4.8	5.0				6.2
Westinghouse Electric	7.7	8.1	6.4	6.7	6.9	6.6	7.3				7.1

Appendix C: Example of Recovery Rate Convergence
(Ijiri, 1979:260)

Recoveries and (Reinvestments)							
	0	1	2	3	4	5	6
Year 0	(1)	0.6	0.72				
1		(0.6)	0.36	0.432			
2			(1.08)	0.648	0.7776		
3				(1.080)	0.6480	0.7776	
4					(1.4256)	0.8554	1.0264
5						(1.6330)	0.9798
6							(2.0062)
Recoveries		0.6	1.08	1.080	1.4256	1.6330	2.0062
Investments		1.0	1.60	1.680	2.1600	2.5056	3.0586
Recovery Rate		0.6	0.675	0.643	0.6600	0.6517	0.6559

The following steps are used to determine an approximation of the theoretical convergent value of the recovery rate:

1. Assumptions
 - a) all recoveries are reinvested;
 - b) each year the firm invests in a typical project which is characterized by a single IRR, economic life, and cash-flow pattern;
2. Sum the total recoveries per year;
3. Divide each year's total recoveries by that year's active investments. An active investment is any investment that has contributed to the cash recovered in any year, such as for year three the active investments were made in years one and two;
4. A plot of the recovery rates would depict wide fluctuations in the early years followed by a convergence to a constant value as the firm matures;
5. The average of the annual recovery rates is considered a viable approximation of the convergent value.

Appendix D: Conditional IRRs, I (Salamon, 1982:297)

Internal Rates of Return													
Life = 20 years				Life = Est Life				Cash Flow				Pattern	
Est		Avg		Ijiri		b=1		b=.8		Ijiri		b=1	
Useful	Real	CRR	ARR	IRR	IRR	IRR	IRR	IRR	IRR	IRR	IRR	IRR	IRR
Life	Grwth	1972-	1972-	(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)
(yrs)	Rate	1978	1978										
Alcoa	20.26	1.7	8.4	4.7	5.6	1.3	0.9	5.6	1.4	1.0			
American Brands	15.47	3.0	10.5	6.1	8.4	3.8	5.0	6.5	2.3	1.8			
American Can	18.98	1.4	8.0	4.9	5.0	0.8	0.1	4.5	0.4	-0.5			
Bethlehem Steel	24.75	0.3	6.9	3.1	3.3	-0.8	-2.0	4.6	0.4	0.6			
Chrysler	26.21	4.0	7.7	1.2	4.5	0.7	-2.6	6.0	2.1	-0.7			
DuPont	13.79	2.4	11.2	8.5	9.3	4.5	7.3	6.5	2.2	2.1			
Esmark	18.04	1.9	11.2	4.8	9.3	4.4	7.8	8.7	4.0	6.4			
General Electric	13.61	4.9	10.5	7.6	8.4	4.1	3.3	5.3	1.3	-0.9			
General Foods	20.86	4.6	11.1	7.4	9.2	4.7	4.9	9.3	4.9	5.3			
General Motors	19.02	3.0	14.3	10.4	13.1	7.6	14.1	12.8	7.5	13.4			
Goodyear	18.87	4.4	8.0	4.4	5.0	1.1	-2.2	4.5	0.8	-2.7			
Gulf	18.94	2.7	11.1	6.5	9.2	4.4	6.7	8.8	4.2	6.0			
IBM	8.74	9.5	18.9	14.1	18.2	12.8	16.7	11.8	7.4	6.6			
Int'l Harvester	23.22	1.2	8.2	4.3	5.3	1.0	0.8	6.1	1.7	2.6			
Int'l Paper	21.75	3.4	11.3	7.1	9.4	4.7	6.5	9.8	5.0	7.4			
RCA	8.76	5.5	13.5	4.8	12.1	7.3	9.5	3.5	-0.3	-2.3			
Texaco	24.34	4.3	10.0	6.7	7.8	3.5	2.6	8.6	4.3	4.3			
United Technology	15.69	6.4	9.0	5.7	6.4	2.6	-1.2	4.4	0.8	-3.2			
U.S. Steel	31.62	-0.1	6.2	4.5	2.1	-1.8	-3.6	4.7	0.4	1.6			
Westinghouse	19.29	3.1	7.1	4.8	3.6	-0.2	-3.5	3.3	-0.4	-3.9			

Appendix E: Observability of the CRR (Stark, 1987b:101)

Stark offers the following unrealistic example to illustrate the incompatibilities between the empirical cash recovery rate (ECRR) and the true cash recovery rate (TCRR).

Investment	Recovery	Recovery
(time $t = 0$)	(time $t = 1$)	(time $t = 2$)
(1000)	774	387

This project has an IRR of 12 percent. The initial investment is composed of 700 in depreciable assets and 300 in stock. The stock balance stands at 150 after one year and zero after two. No other current assets exist. Thus, associated with this pattern of recoveries is a pattern of funds generated from operations (FGFO) of:

FGFO	FGFO
(time $t = 1$)	(time $t = 2$)
624	237
(= 774 - 150)	(= 387 - 150)

Consider a steady state in which the company invests 1000 a year in this project. From the first year of the firm's life onwards, a partial balance sheet would contain the following balances:

Fixed Assets (at cost)	1400
Current Assets	450 (= 300 + 150)

Annual FGFO would stabilize at 861 (= 624 + 237) whereas actual recoveries would stabilize at 1161 (= 774 + 387). From this, it is apparent that the observed, or ECRR, stabilizes at $861/1850 = 0.4654$, whereas the TCRR stabilizes at $1161/2000 = 0.5805$.

Appendix F: Conditional IRRs, II (Gordon and Hamer, 1988:519)

	Useful Life = 20 years						Useful Life = Est Life		
	i=1/3n	i=1/2n	i=2/3n				i=1/3n	i=1/2n	i=2/3n
	B=.867 IRR(7)	B=.905 IRR(8)	B=.926 IRR(9)	IRR(10)	IRR(11)	IRR(12)	IRR(10)	IRR(11)	IRR(12)
Alcoa	1.33	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40
American Brands	3.93	3.80	3.80	2.30	2.40	2.40	2.30	2.40	2.40
American Can	0.80	0.80	0.80	0.40	0.50	0.60	0.40	0.50	0.60
Bethlehem Steel	-0.79	-0.70	-0.60	0.40	0.40	0.40	0.40	0.40	0.40
Chrysler	0.66	0.30	0.30	2.10	2.30	2.40	2.10	2.30	2.40
DuPont	4.62	4.40	4.30	2.10	2.20	2.20	2.10	2.20	2.20
Esmark	4.57	4.30	4.10	4.20	3.90	3.80	4.20	3.90	3.80
General Electric	4.14	4.20	4.30	1.10	1.60	1.80	1.10	1.60	1.80
General Foods	4.76	4.70	4.70	4.90	4.90	4.90	4.90	4.90	4.90
General Motors	7.76	7.20	6.90	7.80	7.10	6.80	7.80	7.10	6.80
Goodyear	1.08	1.40	1.50	0.60	1.00	1.20	0.60	1.00	1.20
Gulf	4.56	4.30	4.20	4.30	4.00	4.00	4.30	4.00	4.00
IBM	12.78	12.40	12.20	7.10	7.50	7.60	7.10	7.50	7.60
International Harvester	1.04	1.00	1.10	1.80	1.70	1.70	1.80	1.70	1.70
International Paper	4.84	4.70	4.60	5.20	4.90	4.80	5.20	4.90	4.80
RCA	7.32	1.70	7.00	-0.80	0.10	0.20	-0.80	0.10	0.20
Texaco	3.53	3.60	3.70	4.30	4.40	4.40	4.30	4.40	4.40
United Technology	2.57	3.00	3.20	0.50	1.20	1.50	0.50	1.20	1.50
U.S. Steel	-1.51	-1.70	-1.60	0.50	0.40	0.40	0.50	0.40	0.40
Westinghouse	-0.25	0.10	0.20	-0.60	0.20	0.00	-0.60	0.20	0.00
Mean	3.37	3.32	3.31	2.48	2.61	2.63	2.48	2.61	2.63

Appendix G: Correlations Between CRR-based Estimates of Conditional IRRs (Griner and Stark, 1988:302-303)

A. Correlations between Lee and Stark CRR-based IRR estimates

	LS1	LS2	LS3	LS4	LS5	LS6	LS7	LS8
LS1	1.000							
LS2	0.971	1.000						
LS3	0.960	0.998	1.000					
LS4	0.980	0.957	0.954	1.000				
LS5	0.981	0.965	0.962	0.999	1.000			
LS6	0.841	0.935	0.947	0.860	0.873	1.000		
LS7	0.942	0.990	0.997	0.946	0.955	0.954	1.000	
LS8	0.898	0.974	0.984	0.908	0.920	0.979	0.990	1.000

B. Correlations between Ijiri CRR-based IRR estimates

	I1	I2	I3	I4	I5	I6	I7	I8
I1	1.000							
I2	0.973	1.000						
I3	0.938	0.991	1.000					
I4	0.983	0.979	0.963	1.000				
I5	0.982	0.984	0.971	0.999	1.000			
I6	0.721	0.850	0.907	0.803	0.818	1.000		
I7	0.875	0.958	0.987	0.921	0.933	0.959	1.000	
I8	0.836	0.936	0.973	0.892	0.905	0.971	0.995	1.000

C. Correlations between Lee and Stark and Ijiri CRR-based IRR estimates

	I1	I2	I3	I4	I5	I6	I7	I8
LS1	0.443	0.460	0.451	0.439	0.443	0.396	0.430	0.419
LS2	0.355	0.408	0.419	0.371	0.379	0.433	0.422	0.426
LS3	0.332	0.395	0.413	0.354	0.363	0.448	0.425	0.432
LS4	0.431	0.461	0.464	0.447	0.451	0.442	0.456	0.451
LS5	0.420	0.455	0.460	0.437	0.442	0.445	0.455	0.451
LS6	0.166	0.260	0.300	0.220	0.231	0.433	0.338	0.364
LS7	0.304	0.378	- .405	0.334	0.344	0.461	0.426	0.437
LS8	0.264	0.350	0.384	0.305	0.316	0.469	0.414	0.432

D. Correlations between Lee and Stark and Ijiri CRR based estimates and Accounting Rates of Return

	LS1	LS2	LS3	LS4	LS5	LS6	LS7	LS8
ARR1	0.429	0.341	0.321	0.405	0.397	0.127	0.297	0.242
ARR2	0.461	0.359	0.339	0.435	0.426	0.125	0.314	0.249
	I1	I2	I3	I4	I5	I6	I7	I8
ARR1	0.679	0.639	0.603	0.637	0.635	0.376	0.551	0.510
ARR2	0.641	0.589	0.545	0.593	0.589	0.306	0.485	0.443

The correlation coefficients presented are Pearson Correlation Coefficients. The same basic picture is portrayed of the levels of correlation between the alternative firm rankings if the Spearman Rank Correlation Coefficient is employed. All the correlation coefficients reported are significantly different from zero at a five percent level of significance.

Appendix H: Regression Results for CF/IE Relationship
(Griner and Stark, 1988:305-306)

Regression of the Ratio of Cash Flow to Capital Expenditures
on Growth and IRR_(est): $(CF/IE = \beta_0 + \beta_1 g + \beta_2 IRR_{(est)} + u)$

Variable	Estimated	Standard	Significance
Constant	2.542	0.415	0.05 ^a
g	-10.182	1.733	0.01 ^b
ARR1	3.872	4.384	0.19 ^c
Adjusted R ² = 0.114			
Constant	2.642	0.420	0.05 ^a
g	-9.958	1.780	0.01 ^b
ARR2	2.021	3.470	0.29 ^c
Adjusted R ² = 0.113			
Constant	0.557	0.250	0.14 ^a
g	-13.291	1.199	0.01 ^b
LS1	15.529	1.144	0.01 ^c
Adjusted R ² = 0.447			
Constant	0.454	0.234	0.05 ^a
g	-10.397	1.108	0.01 ^b
LS2	12.612	0.820	0.01 ^c
Adjusted R ² = 0.500			
Constant	0.392	0.241	0.05 ^a
g	-9.562	1.119	0.01 ^b
LS3	12.059	0.805	0.01 ^c
Adjusted R ² = 0.489			
Constant	0.235	0.273	0.05 ^a
g	-12.574	1.204	0.01 ^b
LS4	16.366	1.248	0.01 ^c
Adjusted R ² = 0.432			
Constant	0.237	0.271	0.05 ^a
g	-12.194	1.193	0.01 ^b
LS5	15.940	1.203	0.01 ^c
Adjusted R ² = 0.437			
Constant	0.073	0.202	0.05 ^a
g	-5.036	0.974	0.01 ^b
LS6	9.199	0.445	0.01 ^c
Adjusted R ² = 0.631			

Constant	0.349	0.250	0.05 ^a
<i>g</i>	-8.666	1.137	0.01 ^b
LS7	11.376	0.788	0.01 ^c
Adjusted $R^2 = 0.473$			
Constant	0.285	0.238	0.05 ^a
<i>g</i>	-7.489	1.101	0.01 ^b
LS8	10.298	0.653	0.01 ^c
Adjusted $R^2 = 0.511$			
Constant	2.345	0.362	0.05 ^a
<i>g</i>	-11.261	1.803	0.01 ^b
I1	6.325	3.517	0.04 ^c
Adjusted $R^2 = 0.121$			
Constant	2.331	0.360	0.05 ^a
<i>g</i>	-10.671	1.617	0.01 ^b
I2	5.833	3.093	0.04 ^c
Adjusted $R^2 = 0.122$			
Constant	2.327	0.356	0.05 ^a
<i>g</i>	-10.370	1.555	0.01 ^b
I3	5.556	2.897	0.03 ^c
Adjusted $R^2 = 0.122$			
Constant	2.167	0.358	0.05 ^a
<i>g</i>	-11.545	1.701	0.01 ^b
I4	8.014	3.224	0.01 ^c
Adjusted $R^2 = 0.129$			
Constant	2.184	0.358	0.05 ^a
<i>g</i>	-11.382	1.680	0.01 ^b
I5	7.724	3.191	0.01 ^c
Adjusted $R^2 = 0.128$			
Constant	2.207	0.339	0.05 ^a
<i>g</i>	-9.266	1.460	0.01 ^b
I6	5.350	2.083	0.01 ^c
Adjusted $R^2 = 0.130$			
Constant	2.334	0.352	0.05 ^a
<i>g</i>	-10.018	1.503	0.01 ^b
I7	5.126	2.657	0.03 ^c
Adjusted $R^2 = 0.122$			

Constant	2.335	0.350	0.05 ^a
<i>g</i>	-9.833	1.485	0.01 ^b
I8	4.868	2.499	0.03 ^c
Adjusted R ² = 0.355			

^aSignificance with respect to a value of 1, not 0.

^bOne-tailed test.

^cOne-tailed test.

Appendix I: 1A Industry Generation Template

PRMTR1A.MCD

CAPT GEISER

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```

* * * * *
*
*   THIS TEMPLATE GENERATES VALUES WHICH DETERMINE THE
*   CASH FLOWS FOR FIRMS WITH A CONSTANT INVESTMENT
*   GROWTH RATE AND WHOSE TYPICAL PROJECT'S CASH-
*   FLOW PROFILE IS DETERMINED BY THE PARAMETER b.
*
* * * * *

```

ORIGIN = 1

PREPARATION

=====

```

n := 5                      * declare economic life *
i := 1 .. n
q := 2                      * declare number of
j := 1 .. q                 "firms"( 9 or less ) *

```

CASH FLOW PROFILE

=====

```

b := .8 + rnd(.3)          * randomize "b" between
j                          .8 and 1., a different
                           value for each firm *
k := 1 .. 31
bprm      := 0             * build matrix of zeroes *
  k,j

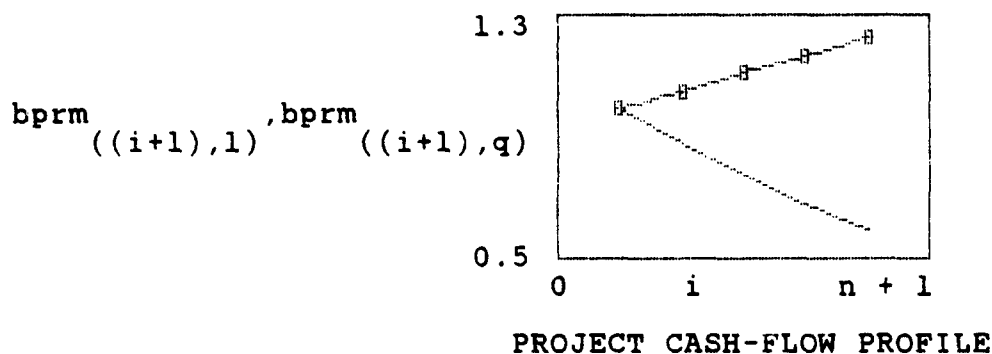
```

YEARLY PARAMATERS

```

bprm      := bi-1          * matrix with yearly
  (i+1),j  j               cash-flow parameters *

```



OUTPUT FILE -----

```

bprm      := b
  1,j      j      * store "b" in cash-
                    flow profile matrix *

```

```

WRITEPRN[B1A_xy PRN] := bprm      * store bprm-matrix
                                   in an ASCII file *

```

INFLATION RATE =====

```

p      := 1      * declare inflationary rate *
  j

```

INVESTMENT GROWTH RATE =====

```

agr      := -.001 + rnd(.096)      * annual growth, based
  j                                     on 5-yr sample period *
agrT =

```

OUTPUT FILE -----

```

g      := 1 + agr      * matrix with growth rates *
  k,j      j

```

```

g      := p      * store "p" in investment
  1,j      j      growth rate matrix *

```

```
WRITEPRN [G1A_xy PRN] := g      * store g-matrix  
                                   in an ASCII file *
```

YEAR 1 RECOVERY PERCENTAGE
=====

```
y1rpi_j := .3 + rnd(.35)      * sets recovery to a min  
                                   of 30%, and a max  
                                   of 65% of investment *
```

```
y1rp_k,j := y1rpi_j          * matrix with year  
                                   1 recovery percentage *
```

```
y1rp_1,j := n                * store "n" in year 1  
                                   recovery percentage matrix *
```

```
WRITEPRN [P1A_xy PRN] := y1rp  * store R-matrix  
                                   in an ASCII file *
```

Appendix J: 1B1 Industry Generation Template

PRMTR1B1.MCD

CAPT GEISER

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```

* * * * *
*
*   THIS TEMPLATE GENERATES VALUES WHICH DETERMINE THE
*   CASH FLOWS FOR FIRMS WITH A VARIABLE INVESTMENT
*   GROWTH RATE AND WHOSE TYPICAL PROJECT'S CASH-
*   FLOW PROFILE IS DETERMINED BY THE PARAMETER b.
*
* * * * *

```

ORIGIN = 1

PREPARATION

=====

```

n := 5                      * declare economic life *
i := 1 .. n

q := 2                      * declare number of
j := 1 .. q                 "firms"( 9 or less ) *

```

CASH-FLOW PROFILE

=====

```

b   := .8 + rnd(.3)        * randomize "b" between
j                                     .8 and 1., a different
                                     value for each firm *

k := 1 .. 31

bprm      := 0              * build matrix of zeroes *
  k,j

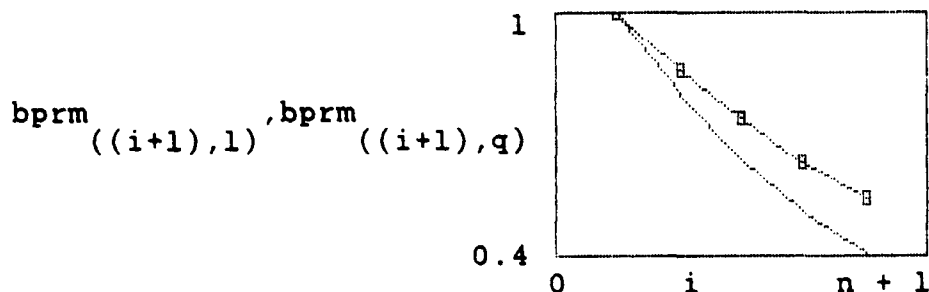
```

YEARLY PARAMATERS

```

bprm      i-1
  (i+1),j := b              * matrix with yearly
j                                     cash-flow parameters *

```



PROJECT CASH-FLOW PROFILE

OUTPUT FILE

```

bprm 1,j := b j      * store "b" in cash-
                        flow profile matrix *

```

```

WRITEPRN[B1B1_xy PRN] := bprm      * store bprm-matrix
                                    in an ASCII file *

```

INFLATION RATE

```

p j := 1      * declare inflationary rate *

```

INVESTMENT GROWTH RATE

```

agr j := -.001 + rnd(.096)      * annual growth, based
                                on 5-yr sample period *

```

```

li j := [1 + agr j]^30      * growth rate for year 30
                                based on sample "agr" *

```

```

* The following Solve Block is used to determine
  the necessary exponents for the base "li" which
  will produce an iterated product that is
  equal to the "agr" compounded for 30 years *

```

```

e1 := .001      e6 := .006      e11 := .011      * initial
e2 := .002      e7 := .007      e12 := .012      guesses *
e3 := .003      e8 := .008      e13 := .013
e4 := .004      e9 := .009      e14 := .014
e5 := .005      e10 := .01      e15 := .015

```

```
inc := .00017
```

```

d  := 2 * inc      * values necessary to solve
1                               simultaneous equations *

```

```
w := 2 ..15
```

```

d  := d      + w * inc      * arrange increments between
w      w-1                the exponents to establish
                               an increasing growth rate *

```

```
Given
```

```
* Solve Block *
```

```
* e1 + e2 + ... + e15 = 1 *
```

```
e1 + e2 + e3 + e4 + e5 + e6 + e7 + e8 + e9 + e10 +
```

```
e1 > 0
```

```
e2 - e1 ≈ d2
```

```
e3 - e2 ≈ d3
```

```
e4 - e3 ≈ d4
```

```
e5 - e4 ≈ d5
```

```
e6 - e5 ≈ d6
```

```
e7 - e6 ≈ d7
```

```
e8 - e7 ≈ d8
```

```
e9 - e8 ≈ d9
```

```
e10 - e9 ≈ d10
```

```
e11 - e10 ≈ d11
```

$$e_{12} - e_{11} \approx d_{12}$$

$$e_{13} - e_{12} \approx d_{13}$$

$$e_{14} - e_{13} \approx d_{14}$$

$$e_{15} - e_{14} \approx d_{15}$$

pwr := Find(e1,e2,e3,e4,e5,e6,e7,e8,e9,e10,e11,e12,e13,
* pwr := Find(e1,...,e15) *

t := 1 ..15

c := 1 ..30

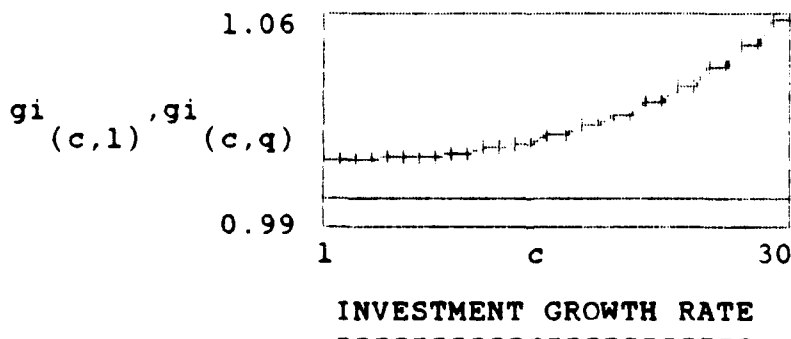
E_{2,t-1} := .5 pwr_t * transform 15
exponents into 30 *

E_{2,t} := .5 pwr_t

gi_{c,j} := li_j^{E_c} * calculate yearly, increasing,
investment growth rates *

\prod_c gi_{c,1} = 0.995 * verify iterated product
of increasing growth rate
produces an overall growth
rate equal to the "agr"
applied to a 30-yr period *

li₁ = 0.995



OUTPUT FILE

```

g      := p      * store "p" in
1,j      j      growth rate matrix *

g      := gi     * matrix with
c+1,j      c,j   increasing growth rates *

WRITEPRN[G1B1_xy PRN] := g      * store g-matrix
                                in an ASCII file *

```

YEAR 1 RECOVERY PERCENTAGE

=====

```

ylrpi  := .3 + rnd(.35) * sets recovery to a min
j                                     of 30%, and a max
                                     of 65% of investment *

ylrp   := ylrpi     * matrix with year
k,j      j          1 recovery percentage *

ylrp   := n      * store "n" in year 1
1,j      recovery percentage matrix *

WRITEPRN[P1B1_xy PRN] := ylrp      * store ylrp-matrix
                                in an ASCII file *

```


Appendix K: 1B2 Industry Generation Template

PRMTR1B2.MCD

CAPT GEISER

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```
* * * * *
*
*   THIS TEMPLATE GENERATES VALUES WHICH DETERMINE THE
*   CASH FLOWS FOR FIRMS WITH A SINUSOIDAL INVESTMENT
*   GROWTH RATE AND WHOSE TYPICAL PROJECT'S CASH-
*   FLOW PROFILE IS DETERMINED BY THE PARAMETER b.
*
* * * * *
```

ORIGIN = 1

PREPARATION

=====

```
      n := 10                      * declare economic life *
      i := 1 .. n

      q := 2                      * declare number of
      j := 1 .. q                  "firms"( 9 or less ) *

      prd := 1                    * "prd" determines number of investment
                                cycles occurring during 30-yr period *
```

CASH-FLOW PROFILE

=====

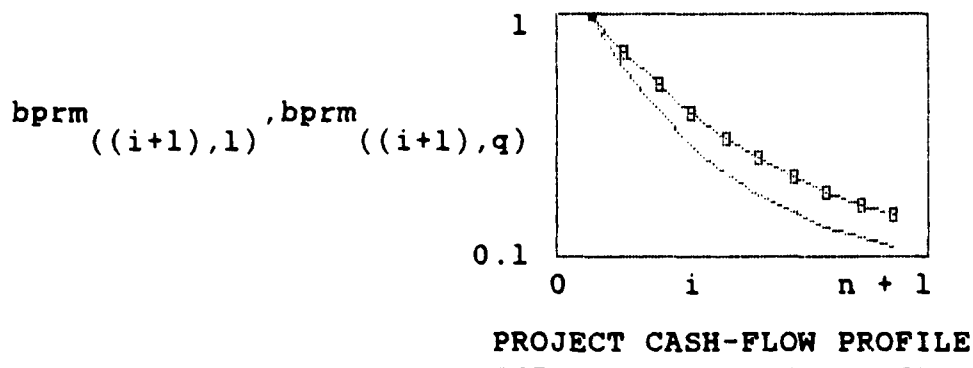
```
      b := .8 + rnd(.3)           * randomize "b" between
      j                                .8 and 1., a different
                                      value for each firm *

      k := 1 .. 31

      bprm      := 0              * build matrix of zeroes *
      k,j
```

YEARLY PARAMATERS

```
      bprm      := bi-1          * matrix with yearly
      (i+1),j      j              cash-flow parameters *
```



OUTPUT FILE -----

```

bprm      := b      * store "b" in cash-
  1,j      j      flow profile matrix *

```

```

WRITEPRN[B1B2_xy PRN] := bprm  * store bprm-matrix
                                in an ASCII file *

```

INFLATION RATE =====

```

p_j := 1      * declare inflationary rate *

```

INVESTMENT GROWTH RATE =====

```

agr_j := -.001 + rnd(.096)  * annual growth, based
                              on 5-yr sample period *

```

```

li_j := [1 + agr_j]^30      * growth rate for year 30
                              based on sample "agr" *

```

```

theta := 0, 2 * pi * prd / 30 .. 2 * pi * prd

```

* The following Solve Block is used to determine the necessary exponents for the base "li" which will produce an iterated product that is equal to the "agr" compounded for 30 years *

Guess: x := .01 * initial guess *

Given

$$\sum_{\theta} x (\alpha + \sin(\theta)) \approx 1$$

ans(α) := find(x)

$\alpha_j := .4 + \text{rnd}(.6)$ * this range of " α " provides possibility for some years to experience negative growth *

coef_j := ans[α_j] * vector of values solved for "x" *

l := 1 .. 30

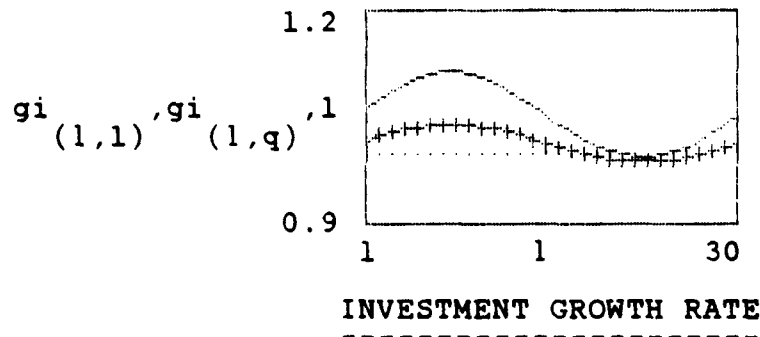
angle_l := l $\cdot \frac{\pi}{15 \text{ prd}}$ * argument for sine function *

exp_{l,j} := coef_j [$\alpha_j + \sin[\text{angle}_l]$] * exponents for "li" *

gi_{l,j} := li_j^{exp_{l,j}} * calculate yearly, sinusoidal, investment growth rates *

$\prod_l \text{gi}_{l,1} = 4.597$ * verify iterated product of sinusoidal growth rate produces an overall growth rate equal to the "agr" applied to a 30-year period *

li₁ = 4.837 * as the increments in " θ ", above, decrease, "li" and iterated product approach



OUTPUT FILE

```

g1,j := pj          * store "p" in growth rate matrix *

g1+1,j := gi1,j      * matrix with sinusoidal
                        growth rates *

WRITEPRN[G1B2_xy PRN] := g      * store g-matrix
                                in an ASCII file *
```

YEAR 1 RECOVERY PERCENTAGE

```

ylrpij := .3 + rnd(.35)  * sets recovery to a min
                        of 30%, and a max
                        of 65% of investment *

ylrpk,j := ylrpij      * matrix with year
                        1 recovery percentage *

ylrp1,j := n            * store "n" in year 1
                        recovery percentage matrix *

WRITEPRN[P1B2_xy PRN] := ylrp  * store ylrp-matrix
                                in an ASCII file *
```

Appendix L: 2A Industry Generation Template

PRMTR2A.MCD

CAPT GEISER

PAGE 1/3

```

* * * * *
*
*   THIS TEMPLATE GENERATES VALUES WHICH DETERMINE
*   THE CASH FLOWS FOR FIRMS WITH A CONSTANT
*   INVESTMENT GROWTH RATE AND WHOSE TYPICAL
*   PROJECT'S CASH-FLOW PROFILE IS CONCAVE.
*
* * * * *

```

ORIGIN = 1

PREPARATION

=====

```

n := 5                      * declare economic life *
i := 1 .. n
q := 2                      * declare number of
j := 1 .. q                  "firms"( 9 or less ) *

```

CASH-FLOW PROFILE

=====

```

B  := .501 + rnd(.498)      * randomize year of
j                                maximum cash inflow *

k := 1 .. 31

bprm := 0                    * build matrix of zeroes *
      k,j

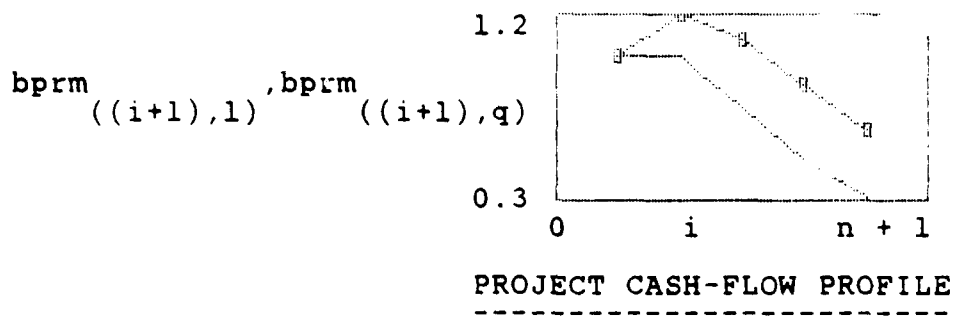
```

YEARLY PARAMATERS

```

bprm      i-1
(i+1),j  := i B_j          * matrix with yearly
                                cash-flow parameters *

```



OUTPUT FILE

```

bprm      := B
1,j      j      * store "B" in cash-
                  flow profile matrix *

WRITEPRN [B2A_xy PRN] := bprm      * store b-matrix
                                   in ASCII file *

```

INFLATION RATE

=====

```

p      := 1
j      * declare inflationary rate *

```

INVESTMENT GROWTH RATE

=====

```

agr      := -.001 + rnd(.096)      * annual growth, based
j      on 5-yr sample period *

agrT =

```

OUTPUT FILE

```

g      := 1 + agr
k,j      j      * matrix with growth rates *

g      := p
1,j      j      * store "p" in investment
                  growth rate matrix *

WRITEPRN [G2A_xy PRN] := g      * store g-matrix
                                   in an ASCII file *

```

YEAR 1 RECOVERY PERCENTAGE
=====

```
ylrpi_j := .3 + rnd(.35)      * sets recovery to a min  
                                of 30%, and a max  
                                of 65% of investment *  
  
ylrp_k,j := ylrpi_j          * matrix with year  
                                1 recovery percentage *  
  
ylrp_1,j := n                * store "n" in year 1  
                                recovery percentage matrix *  
  
WRITEPRN[P2A_xy PRN] := ylrp  * store R-matrix  
                                in an ASCII file *
```

Appendix M: CRR Spreadsheet Excerpt

INPUT:

YEAR	0		1		
	RCVRS	INV	RCVRS	INV	RCVRS
	*	-1	0.5049		0.539233
				-1.03	0.52005

INV		-1		-1.03	
RCVRS			0.5049		1.05928
CRR			0.5049		0.5218
YEAR	0		1		

CALCULATIONS:

YEAR	0	1	2	3	4
n CF	-1	-0.5251	-0.0119	0.40441	0.71059
n IRR	0.36333				
pjt CF	(yr 14)	-2.06611	1.04318	1.11411	0.89223
pjt IRR	(yr 14)	0.339638		(yr 25)	0.339638
n NPV	30.17672	16.77349	9.823566	5.988649	3.78575
pjt NPV	1.660688	1.34085	1.07149	0.84351	0.64967

OUTPUT:

	*	n	n IRR	pjt IRR	b or B
	YEAR	CRR	n NPV	pjt NPV	b'
	*	0	0.339638	0.339638	0.534
	1	0.5049	32.45786	1.660688	1
	2	0.52180	16.77349	1.34085	1.068
	3	0.49274	9.823566	1.07149	0.8553
	4	0.44905	5.988649	0.84351	0.6089

Appendix N: 1A Industry IRR Solution Template

IRR1A.MCD

CAPT GEISER

PAGE 1/3

```

* * * * *
*
*   THIS TEMPLATE CALCULATES IRRs FOR FIRMS WHOSE
*   TYPICAL PROJECT'S CASH FLOWS ARE DETERMINED
*   BY THE CONSTANT PARAMETERS b AND g.
*
* * * * *

```

ORIGIN = 1

INPUT PARAMETERS AND RATES

=====

```

ALLDATA := READPRN[R1A_5A1
                   PRN ]

```

```

rws := rows(ALLDATA)      rws = 31
i := 2 ..rws              j := 1 ..rws - 1

```

```

n := ALLDATA              n = 5
    1,1

```

```

nIRR := ALLDATA           nIRR = 0.2754
    1,2

```

```

pIRR := ALLDATA           pIRR = 0.2755
    1,3

```

```

b := ALLDATA              b = 0.8326
    1,4

```

```

p := ALLDATA              p = 1
    1,5

```

```

g := ALLDATA              g = 1.005
    2,5

```

```

CRR1  := ALLDATA
    i-1      i,1

```

```

nNPV  := ALLDATA
    i-1      i,2

```

```

pNPV  := ALLDATA
    i-1      i,3

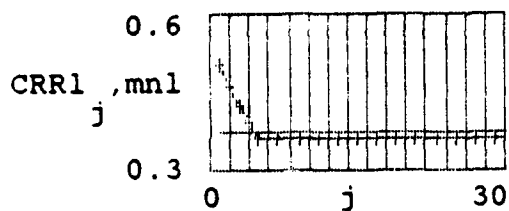
```

CRR ANALYSIS

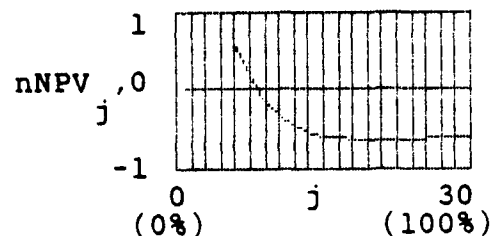
=====

k := 1 .. (rws - 1) - m

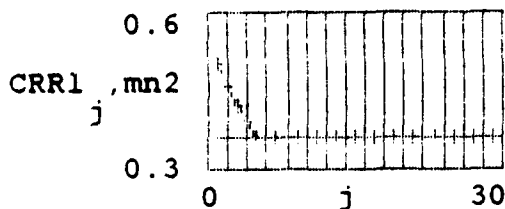
* "m" is declared below *

CRR2_k := CRR1_{k+m}CRR1_{rws-1} = 0.3608CRR2_{(rws-1)-m} = 0.3608mn1 := mean(CRR1)
mn1 = 0.3719mn2 := mean(CRR2)
mn2 = 0.3608

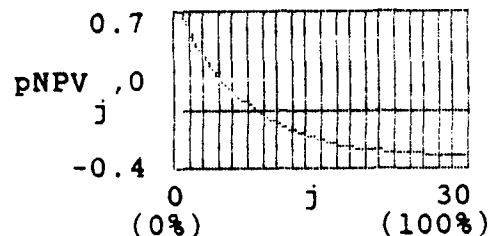
CRR vs YEAR



FIRM NPV vs DR



CRR vs YEAR



PROJECT NPV vs DR

m = 5

* select "m" as first year of
steady state CRR values - these
values are used to calculate "mn2" *

SOLVE FOR IRR

=====

$$\alpha := \frac{(1 - p g) p^n g^n}{1 - p^n g^n}$$

Appendix O: 1B Industry IRR Solution Template

IRR1B.MCD

CAPT GEISER

PAGE 1/4

```
* * * * *
*
*   THIS TEMPLATE CALCULATES IRRs FOR FIRMS WHOSE
*   TYPICAL PROJECT'S CASH FLOWS ARE DETERMINED
*   BY A CONSTANT b AND A VARYING g.
*
* * * * *
```

ORIGIN = 1

INPUT PARAMETERS AND RATES

=====

ALLDATA := READPRN[R1B2T_B7
PRN]

rws := rows(ALLDATA) rws = 31
i := 2 ..rws j := 1 ..rws - 1

n := ALLDATA n = 10
 1,1

nIRR := ALLDATA nIRR = 0.2284
 1,2

pIRR := ALLDATA pIRR = 0.2295
 1,3

b := ALLDATA b = 0.8722
 1,4

p := ALLDATA p = 1
 1,5

CRR1 := ALLDATA
 i-1 i,1

nNPV := ALLDATA
 i-1 i,2

pNPV := ALLDATA
 i-1 i,3

g := ALLDATA
 i-1 i,5

CRR ANALYSIS

=====

k := 1 ..(rws - 1) - m * "m" is declared below *

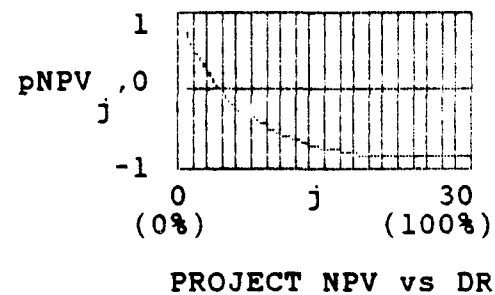
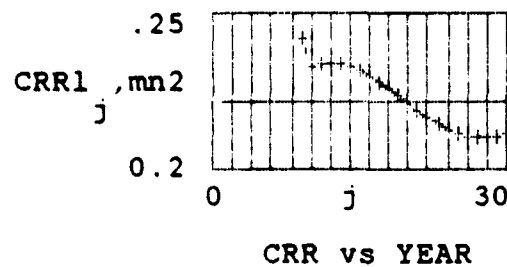
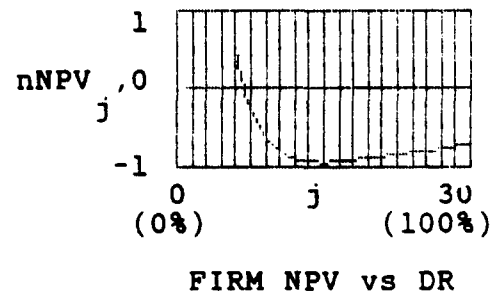
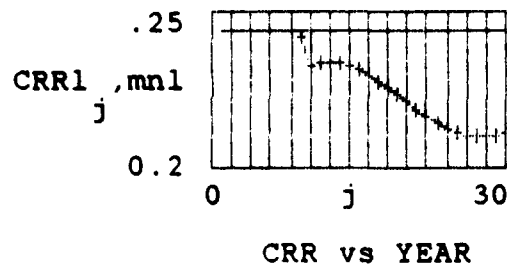
CRR2_k := CRR1_{k+m}

CRR1_{rws-1} = 0.2117

CRR2_{(rws-1)-m} = 0.2117

mn1 := mean(CRR1)
mn1 = 0.2441

mn2 := mean(CRR2)
mn2 = 0.2209



m = 10

* select "m" as first year of
steady state CRR values - these
values are used to calculate "mn2" *

SOLVE FOR IRR

=====

CRR := mn2

* declare the average CRR,
"mn2", as the CRR the firm's
cash flows converge upon *

Guess value: $r := pIRR + 1$ * initial guess *

Given

$$CRR \approx \alpha \beta \left[\frac{r^n (r - b)}{r^n - b^n} \right]$$

$IRR(CRR, \alpha, \beta, n, b) := Find(r)$

$$\alpha_j := \frac{[1 - p g_j^n] p g_j^n}{1 - p g_j^n}$$

$$\beta_j := \frac{g_j^n - b^n}{g_j^n [g_j^n - b^n]}$$

$s := 17$ * declare first year, "s", and
 $ny := 5$ duration (years) "ny", of period
 for which IRR is being solved *
 $l := 1 .. ny$

$IRRS_l := IRR[CRR, \alpha_{l+s-1}, \beta_{l+s-1}, n, b]$

$rprm := mean(IRRS) - 1$ * CRR-derived real IRR *

```

* * * * *
*           *
*   rprm = 0.2454   *
*   nIRR = 0.2284   *
*   pIRR = 0.2295   *
*           *
* * * * *

```

```
irrs1 := rprm      irrs2 := nIRR      irrs3 := pIRR

APPENDPRN[11Bx_ylz  ] := irrsT      * append all IRRs
      DAT ]                      an ASCII file *
```

Appendix P: 2B Industry IRR Solution Template

IRR2B.MCD

CAPT GEISER

PAGE 1/4

```

* * * * *
*
*   THIS TEMPLATE CALCULATES IRRs FOR FIRMS WHOSE
*   TYPICAL PROJECT'S CASH FLOWS ARE DETERMINED
*   BY A CONSTANT B AND A VARYING g.
*
* * * * *

```

ORIGIN = 1

INPUT PARAMETERS AND RATES

=====

```

ALLDATA := READPRN[R2B1F_D9
                    PRN ]

rws := rows(ALLDATA)      rws = 31
i := 2 ..rws              j := 1 ..rws - 1

n := ALLDATA              n = 15
    1,1

nIRR := ALLDATA           nIRR = 0.2043
    1,2

pIRR := ALLDATA           pIRR = 0.2089
    1,3

B := ALLDATA              B = 0.5436
    1,4

p := ALLDATA              p = 1
    1,5

CRR1    := ALLDATA
    i-1      i,1

nNPV    := ALLDATA
    i-1      i,2

pNPV    := ALLDATA
    i-1      i,3

g       := ALLDATA
    i-1      i,5

```


CRR ANALYSIS

=====

k := 1 .. (rws - 1) - m * "m" is declared below *

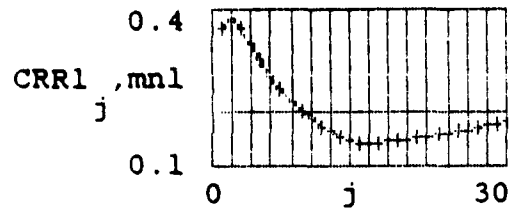
CRR2_k := CRR1_{k+m}

CRR1_{rws-1} = 0.1877

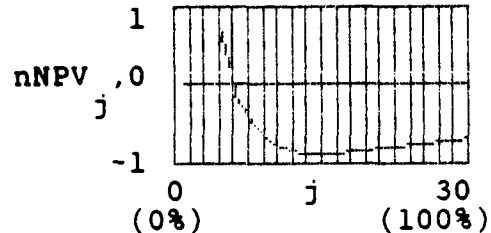
CRR2_{(rws-1)-m} = 0.1877

mn1 := mean(CRR1)
mn1 = 0.2029

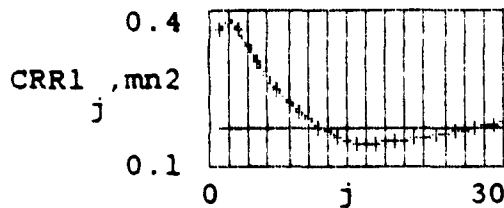
mn2 := mean(CRR2)
mn2 = 0.1737



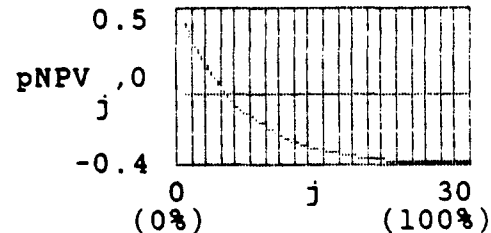
CRR vs YEAR



FIRM NPV vs DR



CRR vs YEAR



PROJECT NPV vs DR

m ≡ 5

* select "m" as first year of
steady state CRR values - these
values are used to calculate "mn2" *

SOLVE FOR IRR

=====

CRR := mn2

* declare the average CRR,
"mn2", as the CRR the firm's
cash flows converge upon *

Guess value: $r := pIRR + 1$ * initial guess *

Given

$$CRR \approx \alpha \cdot \frac{\left[\frac{g^n - B^n}{g^{n-1} (g - B)} - \frac{n \cdot B^n}{g^n} \right]}{\left[\frac{r^n - B^n}{r^{n-1} (r - B)} - \frac{n \cdot B^n}{r^n} \right]}$$

IRR(CRR, α , g , B , n) := Find(r)

$$\alpha_j := \frac{\left[\frac{1 - p \cdot g_j^n}{1 - p \cdot g_j} \right] \cdot p \cdot g_j^n}{1 - p \cdot g_j^n}$$

$s := 17$ * declare first year, "s", and
 $ny := 5$ duration (years) "ny", of period
 for which IRR is being solved *
 $l := 1 \dots ny$

IRRS_l := IRR[CRR, α_{l+s-1} , g_{l+s-1} , B , n]

rprm := mean(IRRS) - 1 * CRR-derived real IRR *

```

* * * * *
*
*   rprm = 0.2345
*   nIRR = 0.2043
*   pIRR = 0.2089
*
* * * * *

```

```
irrs_1 := rprm      irrs_2 := nIRR      irrs_3 := pIRR

APPENDPRN[I2Bx_ylz  ] := irrs^      * append all IRRs
      DAT ]                      an ASCII file *
```

Bibliography

- Anonymous. "Capital Budgeting - Funds Invested Today Determine the Company's Tomorrow," Small Business Report, 10: 52-54 (February 1985).
- Auerbach, Alan J. "Weighted-Average Discount Rates," Modern Developments in Public Finance, edited by Michael J. Boskin. Oxford, United Kingdom: Basil Blackwell Limited, 1987.
- Baumol, William J. "On the Social Rate of Discount," American Economic Review, 112: 788-802 (September 1968).
- Beaves, Robert G. "Net Present Value and Rate of Return: Implicit and Explicit Reinvestment Assumptions," Engineering Economist, 33: 275-302 (Summer 1988).
- Blazouske, J. D. and others. "Current Capital Budgeting Practices in Canada," The Management Accounting Magazine, 62: 51 (March 1988).
- Bradford, David F. "Constraints on Government Investment Opportunities and the Choice of Discount Rate," American Economic Review, 65: 887-899 (December 1975).
- Brief, R. P. "Limitations of Using The Cash Recovery Rate to Estimate the Internal Rate of Return: A Note," Journal of Business Finance and Accounting, 12: 473-475 (Autumn 1985).
- Cannaday, Roger E. and others. "Relevant and Irrelevant Internal Rates of Return," The Engineering Economist, 32: 17-38 (Fall 1986).
- Christensen, David S., Ph.D., Associate Professor of Accounting. Personal Interviews. School of Systems and Logistics, Air Force Institute of Technology, Wright-Patterson AFB OH, 15 August 1990 through 24 August 1991.
- Department of the Air Force. Cost Analysis: Economic Analysis and Program Evaluation for Resource Management. AFR 173-15. Washington: HQ USAF, 4 March 1988.
- Department of Defense. Economic Analysis and Program Evaluation for Resource Management. DoD Instruction 7041.3. Washington: Government Printing Office, 18 October 1972.

- Gordon, L. A. and M. M. Hamer. "Rates of Return and Cash Flow Profiles - An Extension," *Accounting Review*, 63: 514-521 (June 1988).
- Griner, E. H. and A. W. Stark. "Cash Recovery Rates, Accounting Rates of Return, and the Estimation of Economic Performance," *Journal of Accounting and Public Policy*, 7: 293-311 (Winter 1988).
- Hartley, Ronald V. "Inflation, Maintenance of Capital, and the IRR Model: An Appraisal and Alternative," *Decision Sciences*, 19: 93-103 (Winter 1988).
- Hellings, J. "The Cost of Capital in the Real World," *Management Accounting*, 62: 18-20 (May 1984).
- Hsiao, Frank S. T. and W James Smith. "An Analytical Approach to Sensitivity Analysis of the Internal Rate of Return Model," *The Journal of Finance*, 33: 645-649 (May 1978).
- Ijiri, Yuji. "On the Convergence of Periodic Reinvestments by an Amount Equal to Depreciation," *Management Science*, 13: 321-335 (January 1967).
- . "Cash-flow Accounting and Its Structure," *Journal of Accounting, Auditing and Finance*, 1: 331-343 (Summer 1978).
- . "Convergence of Cash Recovery Rates," *Quantitative Planning and Control*. New York: Academic Press, 1979.
- . "Recovery Rate and Cash Flow Accounting," *Financial Executive*, 42: 54-60 (March 1980).
- Ismail, B. E. "Some Time Series Properties of Corporate Cash Recovery Rates," *Contemporary Accounting Research*, 34: 76-88 (Fall 1987).
- Kaplan, Robert S. *Advanced Management Accounting*. New Jersey: Prentice-Hall, Incorporated, 1982.
- Lee, T. A. and A. W. Stark. "Ijiri's Cash Flow Accounting and Capital Budgeting," *Accounting and Business Research*, 68: 125-131 (Spring 1987).
- Lohmann, Jack R. "The IRR, NPV and the Fallacy of the Reinvestment Rate Assumptions," *Engineering Economist*, 33: 303-330 (Summer 1988).

- Livingstone, John Leslie. "Accelerated Depreciation, Tax Allocation, and Cyclical Asset Expenditures of Large Manufacturing Companies," *Journal of Accounting Research*, 6: 245-257 (Autumn 1969).
- Lyon, Randolph M. "Federal Discount Rate Policy, the Shadow Price of Capital, and Challenges for Reforms," *Journal of Environmental Economics and Management*, 18 Part 2: S-29-S-50 (March 1990).
- MathCAD Analytical Software, Version 2.5. MathSoft, Inc., One Kendall Square, Cambridge MA: Copyright 1986-1989.
- McIntosh and others. "Capital Budgeting Practices of Corporate Real Estate Executives," *Appraisal Journal*, 55: 125-137 (January 1987).
- Moore, James S. and Alan K. Reichert. "A Multivariate Study of Firm Performance and the Use of Modern Analytical Tools and Financial Techniques," *Interfaces*, 19: 79-87 (May/June 1989).
- Moore, W. T. and S. Chen. "Implementing the IRR Criterion When Cash Flow Parameters Are Unknown," *Financial Review*, 19: 351-358 (November 1984).
- Mukherjee, Tarun K. "The Capital Budgeting Process of Large U.S. Firms; An Analysis of Capital Budgeting Manuals," *Managerial Finance*, 14: 28-35 (Winter 1988).
- Newbery, David M. and others. "The Isolation Paradox and the Discount Rate for Benefit-Cost Analysis: A Comment; The Isolation Paradox; the Social Discount Rate; and Neutrality: Further Thoughts," *Quarterly Journal of Economics*, 105: 235-241 (February 1990).
- Office of Management and Budget. *Discount Rates to Be Used in Evaluating Time-Distributed Costs and Benefits*. OMB Circular No. A-94. Washington: Government Printing Office, 1972.
- Pike, Richard H. "The Capital Budgeting Revolution," *Management Accounting*, 66: 28-30 (October 1988).
- QUATTRO PRO Spreadsheet Software, Version 2.0. Borland International, Inc., 1800 Green Hills Road, Scotts Valley CA: Copyright 1987-1990.
- Quirk, James P. and Katsuaki L. Terasawa. *The Choice of Discount Rate Applicable to Government Resource Use*. R-3464-PA&E. Santa Monica CA: Rand Corporation, December 1987.

Salamon, Gerald L. "Models of the Relationship Between the Accounting and Internal Rate of Return: An Examination of the Methodology," *Journal of Accounting Research*, 11: 296-303 (Autumn 1973).

----- . "Cash Recovery Rates and Measures of Firm Profitability," *The Accounting Review*, 57: 292-302 (April 1982).

----- . "Accounting Rates of Return," *The American Economic Review*, 75: 495-504 (June 1985).

----- . "On the Validity of Accounting Rates of Return in Cross-sectional Analysis: Theory, Evidence, and Implications," *Journal of Accounting and Public Policy*, 7: 267-292 (Winter 1988).

Schmidgall, Raymond S. and James Damitio. "Current Capital Budgeting Practices of Major Lodging Chains," *Real Estate Review*, 20: 40-45 (Fall 1990).

Seagraves, J. A. "More on the Social Rate of Discount," *Quarterly Journal of Economics*, 84: 430-450 (August 1970).

Shishko, Robert. *Choosing the Discount Rate for Defense Decision Making*. R-1953-RC. Santa Monica CA: Rand Corporation, July 1976.

----- . *The Discount Rate for Defense Decisionmaking: Some New Considerations*. P-6227. Santa Monica CA: Rand Corporation, November 1978.

Stark, Andrew W. "The Cash Recovery Rate Approach to the Estimation of Economic Performance," *The Fifth Tom Robertson Memorial Lecture*. University of Edinburgh, Essex, United Kingdom, January 1987a.

----- . "On the Observability of the Cash Recovery Rate," *Journal of Business and Accounting*, 32: 99-108 (Spring 1987b).

----- . "On Testing the Assumptions of the Cash Recovery Rate Approach to Estimation of Economic Performance," *Accounting and Business Research*, 19: 277-285 (Summer 1989).

Statistix, Version 3.1. *Analytical Software*, PO Box 130204, Saint Paul MN: Copyright 1988-1990.

Stauffer, Thomas R. "The Measurement of Corporate Rates of Return: A Generalized Formulation," *The Bell Journal of Economics and Management Science*, 2: 434-469 (Autumn 1971).

Stockfisch, Jacob. *Measuring the Opportunity Cost of Government Investment*. Research Paper P-490. Arlington VA: Institute for Defense Analyses, March 1969.

Subcommittee on Economy in Government, Joint Economic Committee. *Economic Analysis of Public Investment Decisions: Interest Rate Policy and Discounting Analysis*. Hearing, 90th Congress, 1st Session, 1968. Washington: Government Printing Office, 1968.

Sweeney, Larry E. and Krishna G. Mantripragada. "Ranking Mutually Exclusive Investments: A Modified Internal Rate of Return Approach," *Akron Business & Economic Review*, 18: 19-25 (Spring 1987).

Tresch, R. *Public Finance*. New York: Business Publications, Incorporated, 1981.

Wright, Brian D. and Peter G. Warr. "The Isolation Paradox, The Social Discount Rate, and Neutrality: Further Thoughts," *The Quarterly Journal of Economics*, 105: 239-241 (February 1990).

Vita

Captain Francis J. Geiser, III, was born in Alexandria, Virginia. He graduated from Antilles High School, Fort Buchanan, Bayamon, Puerto Rico and enlisted in the United States Air Force. While serving as an Airborne Voice Processing Specialist in the 6988th Electronic Security Squadron, Electronic Security Command, RAF Mildenhall, United Kingdom, he entered the Airman Scholarship and Commissioning Program to attend Embry-Riddle Aeronautical University, Daytona Beach, Florida, in pursuit of a Baccalaureate of Science in Aeronautical Engineering. Upon graduating Cum Laude, he received a reserve commission in the USAF and served his first tour as an Aeronautical Engineer in the Flight Dynamics Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson AFB, Ohio. There he was responsible for several wind tunnel and water tunnel test programs until entering the Systems Management Program in the School of Systems and Logistics, Air Force Institute of Technology, in May of 1990.

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13. ABSTRACT (Maximum 200 words) This study explored the potential application of the cash recovery rate in determining the social cost of capital. It specifically investigated the results of using cash recovery rate-based, internal rate of return-estimating relationships, which were formulated under assumptions of constant investment growth rate, to estimate internal rates of return for simulated firm-level financial data generated by using both exponentially increasing and sinusoidal investment growth rates. An extensive literature review is used to build an analytical link between the need for updating the social discount rate, capital budgeting decisions based on internal rates of return, and the proposed behavior of the cash recovery rate as a surrogate for the internal rate of return. Analysis of the estimated internal rates of return, based on an algorithm developed by the author, indicates the relationships remain valid under the more complex growth patterns. This result further supports the use of cash recovery rates as a means of measuring firm profitability and also strengthens the case for additional research into applying an opportunity cost of capital approach for the selection of a social discount rate based on a nationally averaged internal rate of return.				
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